On the stability of contact discontinuity separating two hypersonic sources

A. V. Myasnikov and N. A. Belov
Institute for problems in mechanics, Moscow, Russia

Abstract. We present the results of direct high resolution numerical modelling of instability of contact discontinuity, separating two hypersonic spatial or plane sources. We found that the used numerical scheme allows to see instability in steady state inviscid non heatconductive flow. The instability development significantly depends on grid resolution, but, at the same, it is in good accordance with the linear theory predictions. When a real dissipative process is taken into account (namely, non-linear thermal conduction), the dependents of unstable solution on the grid resolution dissapears. The last makes it possible to study the physical instability correctly and, in particular, to determine the instability type. The influence of CD curvature on its stability is also discussed which allows to extend the application of the results on wide range astrophysical objects.

Keywords: colliding winds, instabilities, thermal conduction.

1. Introduction

The problem of the steady-state interaction of two supersonic flows was studied in detail Lebedev & Myasnikov (1988, 1990) within the framework of the ideal perfect gas model. Stevens et al. (1992) were the first who studied numerically the stability properties of tangential discontinuity in the source flow interaction region. Their results, however, were in disagreement with predictions of linear theory developed by Belov (1997ab) for two-dimensional flows with a stagnation point. On the contrary, the results obtained by Myasnikov et al. (1997) were in accordance with analytical theory. Since the gasdynamic properties of the source-flow interaction region are not only of hydrodynamic but also of a certain astrophysical interest (Myasnikov & Zhekov, 1991, 1993; Stevens et al., 1992; Zhekov et al., 1994, and others), the study of the discrepancy has both theoretical and practical importance. Belov & Myasnikov (1999) have carried out the study in detail for the case of the sources with equal mass loss rates, where contact discontinuity is plane. In the present paper we summarize their results and also go ahead by taking into account the influence of the tangential discontinuity curvature on its stability.

2. Formulation of the problem

It was shown by Lebedev & Myasnikov (1988, 1990) that in the hypersonic limit, and at fixed values of the adiabatic indexes of the two flows, the solution of the problem is completely determined by two dimensional parameters only. Namely, these parameters are the ratio of dynamical pressures of the sources Λ and the critical velocity ratio χ. In this case, the geometrical pattern of the flow depends only on Λ, while the effect of the χ parameter in the steady state solution reduces to the renormalization of the velocity and density fields corresponding to the solution for $\chi = 1$.

The property of the steady-state solution renormalization makes it possible to formulate correctly the problem of the instability of the steady-state solution as follows. Let us have a calculated numerically solution corresponding for a certain Λ and $\chi = 1$ and let a certain $\chi \neq 1$ be specified. We will consider the solution renormalized as mentioned above as initial conditions for obtaining a nonstationary solution of the problem. The aim of the study is to determine the response of the steady-state solution to the numerically generated small perturbations.

The unsteady solution was investigated using the developed by Godunov et al. (1979) *soft fitting* technique on the base of Godunov (1959) scheme, which allows both the fitting and capturing of all flow singularities. In these calculations, only the shocks were fitted, while the contact discontinuity is treated by capturing. The original Godunov scheme is of first order accuracy in space and in time. To increase the resolution properties of the scheme, the ENO-like preproceeding procedure suggested by Sawada (1991) was applied in the present study. We performed the calculations in r, z coordinate system (which is cylindrical or Cartezian for the cases when two spherically symmetric or plane-parallel sources interacts, respectively) and used $M \times N$ grid formed by segments of N fixed straight lines $r = r_n = (n-1)\Delta r$, $\Delta r = \text{const}$, $(n = 1, ..., N)$ with M grid points spaced uniformly between the shocks.

3. Results of numerical modelling

We first considered the solution of the problem for the case of interaction of two spherical symmetric sources with $\Lambda = 1$ and $\chi = 0.5$. The calculations show that the unsteady flow pattern depends significantly on the chosen grid parameters. It turned on that the tangential discontinuity is stable for the grid $M \times N = 90 \times 150$. Although at small times the numerical solution responds to finite disturbances in-

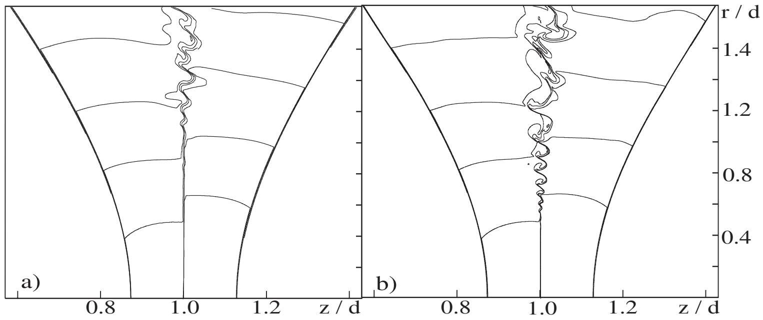

Figure 1. Logarithmic density contours at $t = 18$ for the case with $\Lambda = 1$, $\chi = 0.5$. The grid parameters $M \times N$ equal to 90×300 (a) and 180×600 (b).

troduced by the renormalization, later the disturbances are convected downstream, so at $t = 2$, where t is dimensionless time, the solution is already stable with respect to small perturbations introduced by numerical scheme. The results obtained on finer grids (Figure 1) show that the initial disturbance introduced by the renormalization is again rapidly carried away from the computational region; however, in these cases the grid resolution is sufficient to detect the steady-state solution response to small numerical perturbations. It should be noted, that in all the variants calculated the stagnation point is immovable, so that the contact surface perturbation amplitude is noticeable only starting from a certain distance $r = r_s$ from the stagnation point. This distance decreases when the grid resolution in r or z direction increases.

Keeping $\Lambda = 1$ and the grid resolution the same as in the case presented in Figure 1a, we also considered the interaction of two spherical symmetric sources $\chi = 0.1$, and the interaction of two plane-parallel sources with $\chi = 0.5$. In the both cases we fond the instability development to be more pronounced that in the case presented in Figure 1a. In particular, r_s is smaller, although the stagnation point itself is still immovable. These properties of numerically unstable solutions are in agreement with the exact results of linear theory (Belov, 1997ab), which are valid in the vicinity of the stagnation point (see Belov and Myasnikov, 1999 for more detail) but contradict to results obtained by Stevens et al. (1992). Namely, these authors found a solution where the stagnation point is unstable itself, and they also claimed that the axisymmetric case is more unstable than the planar one.

In order to determine which solution is more reliable we note, firstly, that dealing with the Euler equations one faces with physical incorrectness of the results of the linear stability analysis. Namely, the perturbation growth rate strives to infinity when the wavelength diminishes, whereas in a real flow short wavelength perturbations are always damped by some real physical process. The introduction of such a process into the problem formulation makes it possible to eliminate the incorrectness mentioned above. Such a problem has not been solved yet analytically for 2D flows with a stagnation point. At the same time, there are several studies for classical 1D flows with a tangential discontinuity (Kulikovskii and Shikina, 1997; Shikina, 1987; Ruderman, 2000) which results indicate not only the possibility to eliminate the incorrectness, but also to determine the instability type. In turn, the absolute instability criteria depend on a particular physical process. Secondly, each numerical scheme implementing the Euler equations has its own short-wave instability dumping mechanism which apparently establishes its own criterion of absolute instability.

Then if a scheme produces convective instability (as it happens with our scheme) only scheme-generated convected perturbations should be realized in the vicinity of the stagnation point in accordance with linear theory. If the scheme produces absolute instability, then the solution does not correspond obligatory to linear theory. Such situation takes place, as we believe, with the scheme used by Stevens et al. (1992).

Our scheme, moreover, has the advantage caused with the dependence of the solution on grid resolution. As far as the grid resolution increases the numerical solution tends to the exact solution of Euler equations. If some physical dissipative process is taken into account, one can expect that the numerical dissipation will be negligible when compared with the physical one and numerical solution, being independent on grid resolution increase, corresponds to the exact solution of the problem.

To verify the last statement, we considered a model of an inviscid gas with nonlinear heat conduction (see Myasnikov and Zhekov, 1998 for description of the model in detail) with small thermal conductivity. In this case conduction manifests itself mainly in slight spreading out of density, temperature and tangential velocity fields in the vicinity of contact discontinuity (Figure 2). The flow pattern hardly changes now following a fourfold increase in the grid resolution (Figure 3), unlike the ideal gas flow. Thus, the solution obtained describes the physical instability of convective type of the steady-state flow.

In all calculations carried out above, the ratio of dynamical pressures was accepted $\Lambda = 1$. We also considered the influence of the curvature of the contact discontinuity on its stability in the cases $\Lambda \neq 1$ in the framework of inviscid non gas model without heat conduction. To make conclusions correct, we compared two solutions corresponding to $\Lambda = 2$ and $\Lambda = 0.5$. Flow patterns in these cases are mirror symmetrical, thus, if the grid parameters are chosen the same, the numerical dissipation in

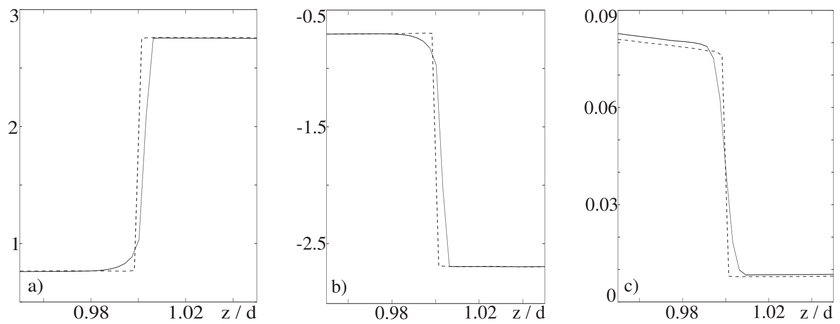

Figure 2. Logarithmic density (a), temperature (b) and r component of velocity (c) distributions in the vicinity of contact discontinuity at $r/d = 0.1$ for $\Lambda = 1$, $\chi = 0.1$. Solid and dashed lines denote the cases without and with thermal conduction respectively. Velocity is normalized by the outflow velocity V_1 densities and pressures - by the quantities $q_1/V_1 d^2$ and $q_1 V_1/d^2$ respectively. The dimensionless thermal conductivity $\Gamma = 0.1$. The grid resolution is the same for both cases.

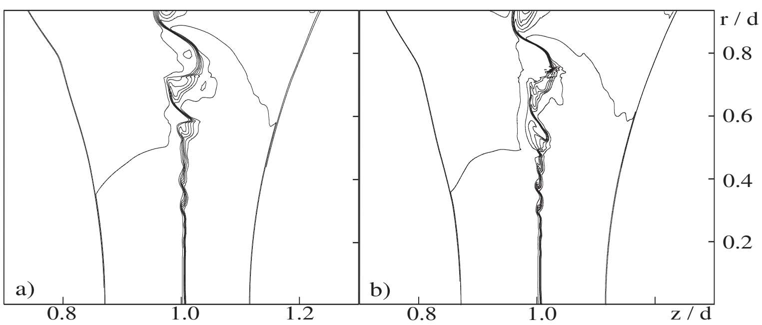

Figure 3. Logarithmic density contours at $t = 18$ for the case with $\Lambda = 1$, $\chi = 0.1$. Thermal conduction is taken into account with dimensional thermal conductivity $\Gamma = 0.1$. The grid parameters $M \times N$ equal to 90×150 (a) and 180×300 (b).

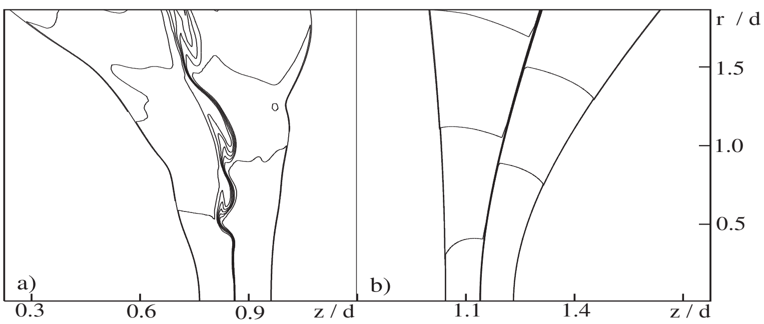

Figure 4. Logarithmic density contours at $t = 18$ for the cases with $\Lambda = 2$ (a) and $\Lambda = 0.5$ (b). The value of $\chi = 0.1$ and the grid parameters ($M \times N = 80 \times 75$) are the same for both cases.

both cases is the same as well. If the value of χ is fixed ($\chi = 0.1$) for both cases, the gas with larger density is from the right of the discontinuity. Numerical results show (Figure 4) that discontinuity is stable for the case $\Lambda = 0.5$, while in the case $\Lambda = 2$ the chosen grid resolution is sufficient to detect the instability development. Further increase of the grid resolution shows the instability development in both cases, but always more pronounced is the case $\Lambda = 2$. This conclusion is in accordance with analytical results obtained by Chalov (1996), although these results are not quite correct in the vicinity of the stagnation point. Further studies, both numerical and analytical, which results can be applied to various astrophysical objects (to colliding stellar winds in WR+O binaries, to the heliopause and others) should be carried out in this direction.

Acknowledgements

This work was supported in part by INTAS-CNES grant 97512, the International Space Science Institute (ISSI) in Bern, and the RFBR Grants 96-01-00738, 98-01-00955.

References

Belov N. A., 1997a, Fluid Dynamics, 32, 219,
Belov N. A., 1997b, Fluid Dynamics, 32, 780
Belov, N.A., Myasnikov, A.V.: 1999, Fluid Dynamics, 34, 379
Chalov, S.V.: 1996, A&A, 308, 995
Gogunov S. K., 1959, Mat.Sbornik, 47, 271
Godunov S. K., Zabrodine A. V., Ivanov M. Ya, Kraiko A. N., Prokopov G. P., 1979, Resolution Numerique des Problems Multidimensionnels de la Dynamique des Gaz, Editions MIR, Moscow
Kulikovsky A. G., Shikina I. S., 1977, Fluid Dynamics, 12, 679
Lebedev M. G., Myasnikov A. V., 1988, in Paskonov and Roslyakov (eds.), Numerical Methods in Aerodynamics, Moscow State University Press, Moscow, p. 3
Lebedev M. G., Myasnikov A. V., 1990, Fluid Dynamics, 25, 629
Myasnikov A.V., Zhekov S.A., 1991, Ap&SS, 184, 287
Myasnikov A.V., Zhekov S.A., 1993, MNRAS, 260, 221
Myasnikov A.V., Zhekov S.A., 1998, MNRAS, 300, 686
Myasnikov A.V., Belov, N.A., Zhekov S.A., 1997, IPM RAS Preprint No 582
Ruderman M.S., 2000, Ap&SS, this issue
Sawada K., 1991, AIAA paper, 1536, 1
Stevens I.R., Blondin J.M., Pollock A.M.T., 1992, ApJ., 386, 265
Shikina I. S., 1987, Fluid Dynamics, 22, 173
Zhekov S.A., Palla F., Myasnikov A.V. 1994, MNRAS, 271, 667