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$\theta \in [0, 1)$ are determined from linear part of (2). Functions $y_j(t, \tau, x, \varepsilon)$ are 2π-periodic on the first two arguments and regularly depend on ε.

From solvability of the boundary problem for the function $y_3(t, \tau, x, \varepsilon)$, it is possible to obtain the normal form, i.e. the system of two special nonlinear complex equations of Schrodinger type. In the simplest case $\sigma = \pi$ the system is reduced to one complex equation of the form

$$4i\frac{\partial \xi}{\partial \tau} = -\frac{\partial^2 \xi}{\partial x^2} - 2i\theta\frac{\partial \xi}{\partial x} + (4\gamma_2 + \theta^2)\xi + 36\beta(|\xi|^2 + 2|\eta|^2)\xi. \tag{4}$$

Here for ξ the periodic boundary conditions $\xi(\tau, x + 2\pi) \equiv \xi(\tau, x)$ are satisfied.

The solutions of this structure contain rapidly oscillating in x components. Thus for the FPU system a special boundary problem of the type of Schrodinger equation is constructed, that plays a role of normal form of boundary problem (2) and describes the behaviour of solutions rapidly oscillating on space variable.

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Traveling-wave solutions in continuous chains of unidirectionally coupled oscillators

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Proposed is a mathematical model of a continuous annular chain of unidirectionally coupled generators given by certain nonlinear advection-type hyperbolic boundary value problem. Such problems are constructed by a limit transition from annular chains of unidirectionally coupled ordinary differential equations with an unbounded increase in the number of links. It is shown that any preassigned finite number of stable periodic motions of the traveling-wave type can coexist in the model.

The mathematical model of an annular chain of unidirectionally coupled generators may be given by a system of ordinary differential equations of the form

$$\dot{u}_j = F(u_j) + mD(u_{j+1} - u_j), \quad j = 1, \ldots, m, \quad u_{m+1} = u_1. \tag{1}$$

where $m \geq 2$, the dot means differentiation with respect to t, $u \in \mathbb{R}^n$, with $n \geq 2$, the vector function $F(u)$ belongs to the class $C^\infty(\mathbb{R}^n; \mathbb{R}^n)$ and D is an $n \times n$ square matrix. It is typically assumed that the partial system

$$\dot{u} = F(u) \tag{2}$$

corresponding to chain (1) has a unique attractor (an equilibrium or a cycle).

Investigating chains of form (1) is relevant in view of various applications. In particular, it is interesting to study the behavior of attractors of such chains as the number m of links increases without bound. Here, we study this problem in the case where partial system (2) has a stable zeroth equilibrium and the coupling matrix D is small in a suitable sense.

For $m \gg 1$, it is totally appropriate to pass from discrete chain (1) to the corresponding continuous model. The transition procedure consists in approximating the variable j/m by a continuous index $x \in [0, 1] (\mathrm{mod}\ 1)$ and replacing the term $m(u_{j+1} - u_j)$ in (1) with the derivative $\partial u/\partial x$. As a result, we obtain the boundary value problem

$$\frac{\partial u}{\partial t} = F(u) + D\frac{\partial u}{\partial x}, \quad u(t, x+1) \equiv u(t, x), \tag{3}$$

which is a mathematical model of a continuous annular chain of unidirectionally coupled generators. We consider problem (3) as an evolutionary equation in a Banach space E consisting of vector functions $u(x)$ of class $\mathrm{W}_2^1([0, 1]; \mathbb{R}^n)$ that are periodic with period 1. We introduce a closed unbounded operator $L = Dd/dx : E \to E$ with a domain $E(L)$ dense in E. The following statement holds.

Lemma. *The operator L is a generator of the semigroup* $\exp(Lt), t \geq 0$, *of class C_0 linear operators bounded in E if and only if the entire spectrum of the matrix D is on the real axis and a basis of its eigenvectors exists in $\mathbb{R}^n$.*

Under some additional restrictions on the matrix D and the character of the nonlinearity of $F(u)$, we consider the problem of attractors (periodic solutions of the traveling-wave type) of boundary value problem (3). By a traveling wave, we mean a periodic solution of boundary value problem (3) of the form $u = u(\theta)$, $\theta = \sigma t + px$, where $\sigma = \mathrm{const} \in \mathbb{R}$ and $p \in \mathbb{Z}$. The vector function $u(\theta) \in \mathbb{R}^n$, $u(\theta + 1) \equiv u(\theta)$, satisfies the ordinary differential equation

$$(\sigma I - pD)\frac{du}{d\theta} = F(u), \tag{4}$$

where I is the unit matrix.

For some special conditions on parameters we show that any preassigned finite number of stable periodic motions of the traveling-wave type can coexist in the model (3).

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