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### Generation of therahertz pulses in a nonlinear dielectric waveguide

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#### ABSTRACT

The generation of broadband terahertz radiation by optical rectification in a gradient planar waveguide is investigated. The conditions for the capture of optical and terahertz pulses and collinear propagation of their bound states are studied. The waveguide structure significantly reduces the diffraction spreading of generated terahertz pulses in comparison to a homogeneous bulk medium.

Keywords: terahertz pulse, optical rectification, gradient waveguide

#### 1. INTRODUCTION

The electromagnetic radiation of terahertz (THz) frequency range presents a unique tool for the study of complex interactions in physical, chemical and biological systems.<sup>1, 2</sup> Appreciable progress has been made in recent decades for development of THz sources.<sup>3–6</sup>

The optical rectification of laser pulses in nonlinear media since its discovery<sup>7</sup> remains one of the most popular and widely used methods for THz generation. THz signal in such scheme is the result of difference-frequency generation within a broad spectrum of single optical pulse. The principal scheme for this effect is the following: an optical photon decays into a terahertz photon and an optical photon with a lower frequency. According to the energy-momentum conservation law for this elementary process one can write:  $\omega(\mathbf{k}) = \omega(\mathbf{k} - \mathbf{q}) + \Omega(\mathbf{q})$ , where  $\omega$  and  $\mathbf{k}$  are the frequency and the wave-vector of the optical photon,  $\Omega$  and  $\mathbf{q}$  are the frequency and the wave-vector of the terahertz photon. Given that  $\Omega \ll \omega$  then  $q \ll k$ . It allows to apply the following expansion  $\omega(\mathbf{k} - \mathbf{q}) = \omega(\mathbf{k}) - \mathbf{q}\partial\omega/\partial\mathbf{k} = \omega(\mathbf{k}) - \mathbf{q}v_g$ , where  $\mathbf{v_g}$  is the vector of optical pulse group velocity. Then one has the Cherenkov condition  $\mathbf{qv_g} = \Omega$  or  $v_g \cos \theta = v_{ph}$ , where  $v_{ph} = \Omega/q$  is the phase velocity of the THz signal,  $\theta$ is the angle between the directions of the propagation of the optical and the THz pulses. In the collinear mode  $(\theta = 0)$  the phase matching condition has the form of the Zakharov - Benney resonance:

$$v_g(\omega) = v_{ph}(\Omega),\tag{1}$$

i.e. the group velocity of the optical pulse is equal to the phase velocity of the THz pulse. The process of photon decay is effective only if that the spectral width  $\Delta \omega$  of the optical pulse covers THz range. For a band-limited signal with a duration  $\tau_p$  one has  $\Delta \omega \sim 1/\tau_p$ . Alternatively  $\Delta \omega \sim \Omega$ , then  $\Omega \tau_p \sim 1$ , i.e. the generated THz pulse is broadband and contains about one period of oscillation.

The phase matching condition (1) is usually very difficult to fulfill as the velocities  $v_g$  and  $v_{ph}$  vary considerably in most dielectric and semiconductor materials. In the non-collinear mode the optical pulse and the THz pulse diverge quickly and the efficiency of generation is insignificant (about  $10^{-6}$  in energy). The method of the tilted wave fronts<sup>8–11</sup> allowed to elevate the efficiency of THz generation up to  $10^{-3} - 10^{-2}$  under cryogenic temperatures.<sup>11</sup> In this method  $\theta$  is the angle between the phase and the beam normal of the optical pulse.

For the needs of THz photonics it is often required to produce robust THz pulses with prescribed spectral or temporal structure. Such possibility is closely connected with solitons in fibers and light bullets in bulk media.

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The problem of light bullets existence in the THz spectral region remains to be weakly explored. In order to get a robust fully spatial localized structure of light in nonlinear medium it is required to imply serious restrictions on the sign of group velocity dispersion and the form of nonlinearity. Few known theoretical results suggest that THz bullet could be coupled with optical one.<sup>12,13</sup> The original research is intended on the further study of such problem in the case of nonlinear dielectric medium with gradient waveguide. The fabrication of special waveguide potentially could fulfill both the requirement for phase matching (1) and the requirement for the bullet robustness.

#### 2. VARIATIONAL APPROACH

The following system of equation can be obtained for the complex envelope  $\psi$  of an optical pulse and the electrical field E of a THz pulse:<sup>5,13</sup>

$$i\frac{\partial\psi}{\partial z} = -\frac{k_2}{2}\frac{\partial^2\psi}{\partial\tau^2} + \alpha E\psi + \omega g_\omega(\mathbf{r}_\perp)\psi - ig_\omega(\mathbf{r}_\perp)\frac{\partial\psi}{\partial\tau} + \frac{c}{2n\omega}\Delta_\perp\psi, \qquad (2)$$

$$\frac{\partial E}{\partial z} = -\beta \frac{\partial}{\partial \tau} \left( |\psi|^2 \right) - g_T(\mathbf{r}_\perp) \frac{\partial E}{\partial \tau} + \frac{c}{2n_T} \Delta_\perp \int_{-\infty}^{\tau} E d\tau' \,. \tag{3}$$

where c is the velocity of light in vacuum, z denotes the direction of propagation set by the waveguide,  $\Delta_{\perp}$  is the transverse Laplace operator,  $\mathbf{r}_{\perp}$  refers to the transverse radius-vector, which originates from the axis of the waveguide,  $\tau = t - z/v_g$ , n and  $n_T$  are the refractive indices on the axis of the waveguide in the optical and the THz ranges,  $\alpha = 4\pi\chi^{(2)}(\omega, 0)\omega/cn_T$ ,  $\beta = 4\pi\chi^{(2)}(\omega, -\omega)/cn_T$ ,  $\chi^{(2)}$  is the second order nonlinear susceptibility,  $k_2$  is the group dispersion coefficient,  $g_{\omega}(\mathbf{r}_{\perp}) = (n^2 - 1)f_{\omega}(\mathbf{r}_{\perp})/2cn$ ,  $g_T(\mathbf{r}_{\perp}) = (n_T^2 - 1)f_T(\mathbf{r}_{\perp})/2cn_T$ ,  $f_{\omega}(\mathbf{r}_{\perp}) = (n^2(\mathbf{r}_{\perp}) - n^2)/(n^2 - 1)$ ,  $f_T(\mathbf{r}_{\perp}) = (n_T^2(\mathbf{r}_{\perp}) - n_T^2)/(n_T^2 - 1)$ ,  $n(\mathbf{r}_{\perp})$  is the optical refractive index, which depends on the transverse coordinate. The waveguide profile meets the following requirement:  $g_{\omega}(\mathbf{r}_{\perp}) = f_{\omega}(\mathbf{r}_{\perp}) = g_T(\mathbf{r}_{\perp}) = f_T(\mathbf{r}_{\perp}) = 0$  on its axis ( $\mathbf{r}_{\perp} = 0$ ).

The analytic study of the Eqs. (2), (3) will be carried out using the variational method of Lagrangian averaging. In the one-dimensional case when  $g_{\omega} = g_T = 0$ ,  $\Delta_{\perp} = 0$  the system of the Eqs. (2) – (3) has the single soliton solution

$$\psi = \frac{|k_2|}{\tau_p} \sqrt{\frac{\Omega}{\alpha\beta}} \exp\left\{i\left[\frac{k_2}{2}\left(\frac{1}{\tau_p^2} - \Omega^2\right) - \Omega\tau\right]\right\} \operatorname{sech}\left(\frac{t - z/v_g}{\tau_p}\right),\tag{4}$$

$$E = -\frac{k_2}{\alpha \tau_p^2} \operatorname{sech}^2\left(\frac{t - z/v}{\tau_p}\right),\tag{5}$$

where

$$\frac{1}{v} = \frac{1}{v_g} - k_2 \Omega \,. \tag{6}$$

This solution posses two arbitrary parameters: pulse temporal duration  $\tau_p$  and red frequency shift  $\Omega$ . The latter is an essential part of highly efficient optical rectification process.<sup>9,10</sup>

The system (2), (3) corresponds to the following Lagrangian

$$L = \frac{i}{2} \left( \psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z} \right) - \frac{k_2}{2} \left| \frac{\partial \psi}{\partial \tau} \right|^2 + \frac{c}{2n\omega} \left| \nabla_\perp \psi \right|^2 - \frac{n^2 - 1}{2cn} \omega f_\omega |\psi|^2 + i \frac{n^2 - 1}{4cn} f_\omega \left( \psi^* \frac{\partial \psi}{\partial \tau} - \psi \frac{\partial \psi^*}{\partial \tau} \right) - \frac{\alpha}{2\beta} \frac{\partial Q}{\partial z} \frac{\partial Q}{\partial \tau} + \frac{\alpha}{\beta} \frac{c}{4n_T} \left( \nabla_\perp Q \right)^2 - \frac{\alpha}{\beta} \frac{n_T^2 - 1}{4cn_T} f_T \left( \frac{\partial Q}{\partial \tau} \right)^2 - \alpha |\psi|^2 \frac{\partial Q}{\partial \tau}, \quad (7)$$

where

$$E = \frac{\partial Q}{\partial \tau}.$$
(8)

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In order to include the transverse coordinate we proceed from (4), (5), (8) to the trial solutions of the form:

$$\psi = k_2 \sqrt{\frac{\Omega}{\alpha\beta}} \rho \exp\left[-i\left(\frac{n\omega}{c}\varphi + \Omega\tau\right)\right] \operatorname{sech}\left[\rho(\tau + k_2\Omega z)\right],\tag{9}$$

$$Q = -\frac{k_2}{\alpha}\rho \tanh\left[\rho(\tau + k_2\Omega z)\right],\tag{10}$$

where  $\rho$  and  $\varphi$  are the unknown functions of coordinate.

Substituting (9), (10) into (7) and integrating on 'fast' variable  $\tau$  we obtain

$$\int_{-\infty}^{+\infty} L d\tau = 2 \frac{k_2^2 n \omega \Omega}{c \alpha \beta} \Lambda \,, \tag{11}$$

where the averaged Lagrangian is

$$\Lambda = \rho \frac{\partial \varphi}{\partial z} + \frac{\rho}{2} (\nabla_{\perp} \varphi)^2 + \frac{ck_2}{2n\omega} \left(\frac{\rho^3}{3} - \Omega^2 \rho\right) - \frac{n^2 - 1}{2n^2} \left(1 - \frac{\Omega}{\omega}\right) f_{\omega} \rho - \frac{n_T^2 - 1}{6nn_T \omega \Omega} f_T \rho^3 + \frac{c^2}{12nn_T \omega \Omega} \left[\frac{\pi^2}{6} - 1 + 2\frac{n_T \Omega}{n\omega} \left(\frac{\pi^2}{12} + 1\right)\right] \frac{(\nabla_{\perp} \rho)}{\rho}.$$
 (12)

In order to exclude inifinite terms arizing due to definition of potential (8) in the diffraction part of (12) we need to apply the following renormalization procedure:<sup>14</sup>

$$\tanh^2 \left[ \rho(\tau + k_2 \Omega z) \right] = 1 - \operatorname{sech}^2 \left[ \rho(\tau + k_2 \Omega z) \right] \to -\operatorname{sech}^2 \left[ \rho(\tau + k_2 \Omega z) \right]$$

Using (12) to write the Euler-Lagrange equations for the functions  $\rho$  and  $\phi$  we get<sup>13–17</sup>

$$\frac{\partial \rho}{\partial z} + \nabla_{\perp}(\rho \nabla_{\perp} \varphi) = 0, \tag{13}$$

$$\frac{\partial\varphi}{\partial z} + \frac{1}{2}(\nabla_{\perp}\varphi)^2 + \frac{ck_2}{2n\omega}\left(\rho^2 - \Omega^2\right) - \frac{n^2 - 1}{2n^2}f_{\omega} - \frac{n_T^2 - 1}{2nn_T\omega\Omega}f_T\rho^2 = \frac{1}{3}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_T\omega\Omega}\frac{1}{\sqrt{\rho}}\Delta_{\perp}\sqrt{\rho}.$$
 (14)

Here we take into account that  $\Omega \ll \omega$ . Because of this the diffraction length of the THz component is significantly lower than the diffraction length of the optical component.

Applying the Madelung transformation we can introduce the complex function

$$\Phi = \sqrt{\rho} \exp\left[\frac{i}{2a} \left(\varphi + \frac{ck_2\Omega^2}{n\omega}z\right)\right].$$
(15)

Then the system (13), (14) transforms into the nonstationary modified Gross-Pitaevskii equation<sup>15</sup>

$$i\frac{\partial\Phi}{\partial z} = -a\Delta_{\perp}\Phi + b(\mathbf{r}_{\perp})|\Phi|^{4}\Phi - g(\mathbf{r}_{\perp})\Phi,$$
(16)

where

$$a = \sqrt{\frac{(\pi^2/6 - 1)}{6nn_T\omega\Omega}}c, \quad b = \frac{c}{4an\omega} \left(k_2 - \frac{n_T^2 - 1}{n_T c\Omega} f_T(\mathbf{r}_\perp)\right), \quad g = \frac{n^2 - 1}{4an^2} f_\omega(\mathbf{r}_\perp). \tag{17}$$

The last term in the right hand side of (16) describes an external 'potential' g – so called trap. As it can be seen from (17) it depends on the profile of the optical waveguide. The THz waveguide makes a contribution to the condensate self-action parameter b.

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Let us consider first the case of planar diffraction in a homogeneous medium with anomalous group dispersion  $(\Delta_{\perp} = 0, k_2 < 0, f_T = f_{\omega} = 0)$ . Then the localized solution of the equation (16) can be derived:

$$\Phi = \left(\frac{3a}{4|b|R_{\perp}^2}\right)^{1/4} \exp\left(i\frac{az}{4R_{\perp}^2}\right) \operatorname{sech}^{1/2}\left(\frac{x}{R_{\perp}}\right).$$
(18)

where  $R_{\perp}$  denotes the transverse size of the optical pulse. From here and from (15) it follows

$$R_{\perp} = \sqrt{\left(\pi^2/6 - 1\right)c\tau_p^2/(2n_T\Omega|k_2|)},\tag{19}$$

$$\rho = \left(\frac{3a}{|b|}\right)^{1/2} \frac{1}{2R_{\perp}} \operatorname{sech}\left(\frac{x}{R_{\perp}}\right),\tag{20}$$

$$\varphi = \left[\frac{1}{12}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_T\Omega R_\perp^2} + \frac{c|k_2|\Omega^2}{n\omega}\right]z.$$
(21)

Given that  $\rho_0 = \sqrt{3a/|b|}/2R_{\perp} = 1/\tau_p$ , where  $\tau_p$  is the duration of the optical pulse at the center of light bullet, we get

$$R_{\perp} = \sqrt{\frac{1}{2} \left(\frac{\pi^2}{6} - 1\right) \frac{n_T}{c\Omega |k_2|}} R_{\parallel}.$$
 (22)

where  $R_{\parallel} = c \tau_p / n_T$  is the longitudinal size of the pulse.

From (22) one also could obtain

$$\Omega = 0.33 \frac{n_T}{c|k_2|} \left(\frac{R_{\parallel}}{R_{\perp}}\right)^2.$$
<sup>(23)</sup>

Thus, the red shift of the carrier frequency increases with the transverse focusing of the formed light bullet.

To check the stability of obtained optical-terahertz bullet we present the solution of the system (13), (14) in the self-similar from:<sup>14, 18</sup>

$$\rho(z,x) = \frac{1}{\tau_p} \frac{R_0}{R_\perp(z)} \operatorname{sech}\left(\frac{x}{R_\perp(z)}\right), \quad \varphi(z,x) = f(z) + \frac{x^2}{2} \frac{R'_\perp(z)}{R_\perp}, \tag{24}$$

where  $R_0$  is the input transverse aperture of the soliton.

For the sake of clarity we consider the simplest case of parabolic waveguide:  $f_{\omega} = \epsilon_{\omega} x^2 / a_{\omega}^2$ ,  $f_T = \epsilon_T x^2 / a_T^2$ , where  $a_{\omega}$  and  $a_T$  are the characteristic scales of the transverse inhomogeneity induced by waveguide in the optical and THz ranges, respectively. Parameters  $\epsilon_{\omega}$  and  $\epsilon_T$  define the signs of refractive index modulation: the waveguide is defocusing under their positive values, and focusing otherwise.

Substituting (24) into (13), (14) we can find in near-axial approximation  $(x^2/R_{\perp}^2 \ll 1)^{14}$ 

$$f' = \frac{ck_2}{2n\omega}\Omega - \left[\frac{ck_2}{2n\omega}\frac{R_0^2}{\tau_p^2} + \frac{1}{6}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_T\omega\Omega}\right]\frac{1}{R_\perp^2},$$
(25)

$$R_{\perp}^{\prime\prime} = -\frac{\partial U}{\partial R_{\perp}},\tag{26}$$

where

$$U = -\epsilon_{\omega} \frac{n^2 - 1}{2n^2 a_{\omega}^2} R_{\perp}^2 - \epsilon_T \frac{n_T^2 - 1}{n n_T \omega \Omega a_T^2} \frac{R_0^2}{\tau_p^2} \ln\left(\frac{R_{\perp}}{R_0}\right) + \frac{c}{n\omega} \left[\frac{k_2 R_0^2}{\tau_p^2} + \frac{1}{2} \left(\frac{\pi^2}{6} - 1\right) \frac{c}{n_T \Omega}\right] \frac{1}{R_{\perp}^2}.$$
 (27)

The equation (26) is an analogue of the second Newton law for a particle with unit mass moving in an external field with the potential energy  $U(R_{\perp})$ . This finding significantly simplifies the further analysis.

Let us consider first the case of homogeneous medium  $(a_{\omega}, a_T \to \infty)$ . Then it follows from (19), that U = 0, which leads to  $R''_{\perp} = 0$  and  $R_{\perp} = R(0) + R'(0)z$ . Any fluctuation of input wavefront curvature of the optical

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pulse could result in  $R'(0) \neq 0$ . Therefore at some sites the focusing will appear (R'(0) < 0), but other ones may be followed with defocusing (R'(0) > 0). Due to this low spatial scale instability a filamentation develops. Therefore, broadly speaking, the considered light bullet is unstable. For normal group velocity dispersion  $(k_2 > 0)$ the expression in the quadratic parentheses of (27) is always positive and this should produce the defocusing in the homogeneous medium.



Figure 1. THz generation in the homogeneous medium (left column) and in the defocusing parabolic waveguide (right column) when the spectrum of optical pulse belongs to the region of anomalous group velocity dispersion.

It follows from (27) that the optical focusing waveguide could form a minimum in 'potential energy'  $U(R_{\perp})$ . The dynamics of parameter  $R_{\perp}$  in the vicinity of this minimum is equivalent to the capture of the generated Thz pulse. The above theoretical estimation does not allow us to make a similar conclusion regarding the THz waveguide.

#### **3. RESULTS OF NUMERICAL SIMULATION**

The theoretical analysis made in the previous section describes only averaged dynamics and unable to catch subtle effects including small-scale instability. In addition, the trial solution is asymptotic and can not describe the initial stage of THz pulse generation when it is absent at the input of a nonlinear crystal.



Figure 2. THz generation in the homogeneous medium (left column) and in the focusing parabolic waveguide (right column) when the spectrum of optical pulse belongs to the region of normal group velocity dispersion.

The numerical simulation of the system (2)) and (3) was performed using conventional RungeKutta integration scheme on the propagation coordinate z. Temporal derivatives in time domain and spatial derivatives on transverse variable x were approximated by high order finite differences. The optical pulse with the envelope function  $\psi = \psi_0$  sechT sechX was launched into a nonlinear crystal, the coordinates were normalized as:  $T = \tau/\tau_p$ ,  $X = x/R_0$ ,  $Z = z\tau_p^2/k_2$ . Electric fields amplitudes were normalized using the following values:  $\psi_m = \sqrt{k_2^2/(\alpha\beta\tau_p^3)}$  and  $E_m = |k_2|/(\alpha\tau_p^2)$ .

Fig. 1 depicts the generation of THz generation with and without defocusing waveguide ( $\epsilon_{\omega}, \epsilon_T > 0$ ) in the spectral region of anomalous optical group velocity dispersion. Both pulses experience self-focusing. Despite the defocusing profile of waveguide the filamentation of the optical pulse appears, which is followed by the distinct transverse splitting of the THz pulse into two parts. This process is in general agreement with the conclusion of the previous section about the instability of the optical-terahertz bullet. Also, the peak intensity of formed filaments is lower than in the case of homogeneous medium.

Fig. 2 illustrates the process of generation in the THz inside the focusing waveguide ( $\epsilon_{\omega}, \epsilon_T < 0$ ) in the spectral region of normal optical group velocity dispersion. In this case generated THz pulse captured by the optical pulse experiences strong curvature of its wave-front accompanied by diffraction broadening. Focusing waveguide captures well enough the backward part of generated THz signal, while the forward part following optical pulse experiences deformation and splitting on filaments. Radiation localization in the focusing waveguide with normal group dispersion is weaker than the localization with anomalous group dispersion. This is somewhat expected because in homogeneous medium with  $k_2 > 0$  the localization does not occur at all.

#### 4. CONCLUSION

Present study shows that the small scale filamentation of an optical-terahertz pulse takes place in a planar waveguide. The theoretical analysis based on the Lagrangian averaging can describe this dynamics only qualitatively. This is particularly true for the generation process when only an optical component is at the input of a nonlinear medium. The agreement of the theory and the numerical results can be better if the test solutions will be chosen more successfully based on the results of the numerical study.

Obtained results suggest that planar focusing waveguide in the case of normal optical group velocity dispersion is able to compensate diffraction broadening of optical and THz pulses at the excess of their profile deformation. Both optical and THz waveguides are required to obtain this regime, but the latter needs to have the strongest impact on the corresponding refractive index. The light bullets in the case of anomalous optical group velocity dispersion are unstable and experience self-focusing in the filamentation regime. Defocusing waveguide can suppress the intensity growth during the self-focusing at the excess of increasing the number of filaments. Further search of optimal form for waveguide profile potentially could improve obtained results and provide better pulse stability. Current progress<sup>19</sup> in multidimensional soliton stabilization by various structures allows us to hope for further success in this direction.

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