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A Waveguide Mode of Generating Terahertz Radiation

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Abstract—An optical technique for generating broadband terahertz radiation in a gradient waveguide is investigated. The phenomena of capturing optical and terahertz pulses in the waveguide are considered. It is shown that capture occurs under conditions of filamentation with both normal and anomalous group dispersions of the optical signal.

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INTRODUCTION

Investigations of different approaches to generating terahertz radiation are becoming more and more popular [1–4], due to possible applications of this radiation in medicine, security systems, image reconstruction, and other fields. The terahertz band is the one least investigated in terms of electromagnetic waves' interaction with matter.

The optical technique of generation in a quadratically nonlinear medium is most efficient. The principle behind it is that each incoming optical photon splits into a terahertz photon and a lower-energy optical photon. According to the law of energy–momentum conservation for this elementary process, we have $\omega(\vec{k}) = \omega(\vec{k} - \vec{q}) + \Omega(\vec{q})$. Here, ω and $\vec{k}$ are the frequency and the wavevector of an optical photon, and Ω and $\vec{q}$ are the frequency and wavevector of a terahertz photon. Since $\Omega \ll \omega$, $q \ll k$. The following expansion into a series is therefore valid: $\omega(\vec{k} - \vec{q}) = \omega(\vec{k}) - \vec{q} \cdot \partial\omega/\partial\vec{k} = \omega(\vec{k}) - \vec{q} \cdot \vec{\upsilon}_g$, where $\vec{\upsilon}_g$ is the group velocity corresponding to the carrier frequency of an optical pulse. We then have Cherenkov condition $\vec{q} \cdot \vec{\upsilon}_g = \Omega$ or $\upsilon_g \cos\theta = \upsilon_{ph}$, where $\upsilon_{ph} = \Omega/q$ is the phase velocity corresponding to a terahertz signal, θ is the angle between the directions of propagation of the optical and terahertz pulses. In the collinear mode ($\theta = 0$), the phase-matching condition takes the form of the Zakharov–Benney resonance; i.e., the group velocity of an optical pulse is equal to the phase velocity of a terahertz signal. The considered photon-splitting process is efficient if width $\delta\omega$ of the optical signal spectrum extends to the terahertz region. For a spectrally limited signal with duration τ_p, we have $\delta\omega \sim 1/\tau_p$. At the same time, $\delta\omega \sim \Omega$, so $\Omega\tau_p \sim 1$; i.e., the generated terahertz pulse is broadband and contains on the order of one period of oscillations.

It is typically very difficult to satisfy the Zakharov–Benney resonance condition, since velocities υ_g and υ_{ph} differ considerably from each other with respect to both dielectrics and semiconductors. In the noncollinear mode of generation, however, optical and terahertz pulses rapidly diverge and the efficiency of generation is very low (on the order of 10^{-6} in terms of energy). The tilted wavefronts of a optical pulse approach has become very popular for increasing the efficiency of generation [5, 6]. In this case, θ is the angle between the normals of an optical signal's phase and ray. As a result, the efficiency of generation rises to 10^{-4}–10^{-3}.

In this work, we study the collinear mode of generation in a gradient focusing waveguide.

ANALYTICAL APPROACH

The following system of equations is derived for optical pulse envelope ψ and electric field E of a terahertz signal:

$$i\frac{\partial\psi}{\partial z} = -\frac{k_2}{2}\frac{\partial^2\psi}{\partial\tau^2} + \alpha E\psi + \omega g_\omega(\vec{r}_\perp)\psi - ig_\omega(\vec{r}_\perp)\frac{\partial\psi}{\partial\tau} + \frac{c}{2n\omega}\Delta_\perp\psi, \tag{1}$$

$$\frac{\partial E}{\partial z} = -\beta\frac{\partial}{\partial\tau}\left(|\psi|^2\right) - g_T(\vec{r}_\perp)\frac{\partial E}{\partial\tau} + \frac{c}{2n_T}\Delta_\perp\int_{-\infty}^{\tau} E d\tau'. \tag{2}$$

Here, c is the speed of light in a vacuum; z is the direction of propagation determined by the waveguide; $\Delta_\perp$ is the transverse Laplacian; $\vec{r}_\perp$ is the transverse radius vector; originating from the waveguide axis, $\tau = t - z/\upsilon_g$; n and n_T are the refractive indices on the waveguide axis in the optical and terahertz regions, respectively; $\alpha = \dfrac{4\pi\chi^{(2)}(\omega,0)\omega}{cn_T}$, $\beta = \dfrac{4\pi\chi^{(2)}(\omega,-\omega)}{cn_T}$,

$\chi^{(2)}$ is the nonlinear second-order susceptibility; k_2 is the group dispersion coefficient; $g_\omega(\vec{r}_\perp) = \dfrac{2\pi}{cn} f_\omega(\vec{r}_\perp)$, $g_T(\vec{r}_\perp) = \dfrac{2\pi}{cn_T} f_T(\vec{r}_\perp)$, $f_\omega(\vec{r}_\perp) = \dfrac{n^2(\vec{r}_\perp) - n^2}{n^2 - 1}$, $f_T(\vec{r}_\perp) = \dfrac{n_T^2(\vec{r}_\perp) - n_T^2}{n_T^2 - 1}$; $n(\vec{r}_\perp)$ is the optical refractive index, which depends on the transverse coordinate; and $n_T(\vec{r}_\perp)$ is the terahertz refractive index, which depends on the transverse coordinate. On the waveguide axis, where $\vec{r}_\perp = 0$, we have $g_\omega(\vec{r}_\perp) = f_\omega(\vec{r}_\perp) = g_T(\vec{r}_\perp) = f_T(\vec{r}_\perp) = 0$.

We performed our analytical study using the averaged Lagrangian approach [7–11].

In the one-dimensional case with $g_\omega = g_T = \Delta_\perp = 0$, system (1), (2) has a one-soliton solution in the form

$$\psi = \frac{|k_2|}{\tau_p}\sqrt{\frac{\Omega}{\alpha\beta}} \times \exp\left\{i\left[\frac{k_2}{2}\left(\frac{1}{\tau_p^2} - \Omega^2\right) - \Omega\tau\right]\right\}\operatorname{sech}\left(\frac{t - z/\upsilon}{\tau_p}\right), \tag{3}$$

$$E = -\frac{k_2}{\alpha\tau_p^2}\operatorname{sech}^2\left(\frac{t - z/\upsilon}{\tau_p}\right), \tag{4}$$

where

$$\frac{1}{\upsilon} = \frac{1}{\upsilon_g} - k_2\Omega. \tag{5}$$

In the general case, system (1), (2) is matched by the Lagrangian

$$L = L_\omega + L_T + L_{\text{int}}, \tag{6}$$

where

$$L_\omega = \frac{i}{2}\left(\psi^*\frac{\partial\psi}{\partial z} - \psi\frac{\partial\psi^*}{\partial z}\right) - \frac{k_2}{2}\left|\frac{\partial\psi}{\partial\tau}\right|^2 + \frac{c}{2n\omega}|\nabla_\perp\psi|^2 - \frac{n^2 - 1}{2cn}\omega f_\omega|\psi|^2 + i\frac{n^2 - 1}{4cn} f_\omega\left(\psi^*\frac{\partial\psi}{\partial\tau} - \psi\frac{\partial\psi^*}{\partial\tau}\right), \tag{7}$$

$$L_T = -\frac{\alpha}{2\beta}\frac{\partial Q}{\partial z}\frac{\partial Q}{\partial\tau} + \frac{\alpha}{\beta}\frac{c}{4n_T}(\nabla_\perp Q)^2 - \frac{\alpha}{\beta}\frac{n_T^2 - 1}{4cn_T} f_T\left(\frac{\partial Q}{\partial\tau}\right)^2, \tag{8}$$

$$L_{\text{int}} = -\alpha|\psi|^2\frac{\partial Q}{\partial\tau}. \tag{9}$$

Here,

$$E = \frac{\partial Q}{\partial\tau}. \tag{10}$$

Proceeding from (3)–(5) and considering (10), we choose trial solutions in order to consider transverse coordinates in the form

$$\psi = k_2\sqrt{\frac{\Omega}{\alpha\beta}}\rho\exp\left[-i\left(\frac{n\omega}{c}\varphi + \Omega\tau\right)\right] \times \operatorname{sech}\left[\rho\left(\tau + k_2\Omega z\right)\right], \tag{11}$$

$$Q = -\frac{k_2}{\alpha}\rho\tanh\left[\rho\left(\tau + k_2\Omega z\right)\right], \tag{12}$$

where ρ and φ are unknown functions of the coordinates.

Substituting (11) and (12) into (7)–(9), after integration with respect to τ we have

$$\int_{-\infty}^{+\infty} L d\tau = 2\frac{k_2^2 n\omega\Omega}{c\alpha\beta}\Lambda, \tag{13}$$

where the averaged Lagrangian

$$\Lambda = \rho\frac{\partial\varphi}{\partial z} + \rho\frac{(\nabla_\perp\varphi)^2}{2} + \frac{ck_2}{2n\omega}\left(\frac{\rho^3}{3} - \Omega^2\rho\right) - \frac{n^2 - 1}{2n^2}\left(1 - \frac{\Omega}{\omega}\right) f_\omega\rho - \frac{n_T^2 - 1}{6nn_T\omega\Omega} f_T\rho^3 + \frac{c^2}{12nn_T\omega\Omega}\left[\frac{\pi^2}{6} - 1 + 2\frac{n_T\Omega}{n\omega}\left(\frac{\pi^2}{12} + 1\right)\right]\frac{(\nabla_\perp\rho)^2}{\rho}. \tag{14}$$

In obtaining the diffraction part of the averaged Lagrangian, we substituted $\tanh^2\left[\rho\left(\tau + k_2\Omega z\right)\right] = 1 - \operatorname{sech}^2\left[\rho\left(\tau + k_2\Omega z\right)\right] \to -\operatorname{sech}^2\left[\rho\left(\tau + k_2\Omega z\right)\right]$ in the diverging term [8].

Writing the Euler–Lagrange equations for ρ and φ with allowance for (22), we arrive at the system of equations for an effective quantum Bose liquid [10, 11]

$$\frac{\partial\rho}{\partial z} + \nabla_\perp(\rho\nabla_\perp\varphi) = 0, \tag{15}$$

$$\frac{\partial\varphi}{\partial z} + \frac{(\nabla_\perp\varphi)^2}{2} + \frac{ck_2}{2n\omega}\left(\rho^2 - \Omega^2\right) - \frac{n^2 - 1}{2n^2} f_\omega - \frac{n_T^2 - 1}{2nn_T\omega\Omega} f_T\rho^2 = \frac{1}{3}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_T\omega\Omega}\frac{\Delta_\perp\sqrt{\rho}}{\sqrt{\rho}}. \tag{16}$$

Here we consider that $\Omega \ll \omega$. The diffraction length of the terahertz component is therefore considerably shorter than that of the optical component.

In performing the Madelung transform [12], we introduce the complex function

$$\Phi = \sqrt{\rho}\exp\left[\frac{i}{2a}\left(\varphi + \frac{ck_2\Omega^2}{n\omega}z\right)\right]. \tag{17}$$

System (15), (16) is then transformed into the nonstationary modified Gross–Pitaevskii equation for a two-dimensional Bose condensate:

$$i\frac{\partial\Phi}{\partial z} = -a\Delta_\perp\Phi + b|\Phi|^4\Phi - g\Phi, \tag{18}$$

where $a = 0.33c/\sqrt{nn_{\mathrm{T}}\omega\Omega}$,

$$b = \frac{c}{4an\omega}\left(k_2 - \frac{n_{\mathrm{T}}^2 - 1}{n_{\mathrm{T}}c\Omega} f_{\mathrm{T}}(\vec{r}_\perp)\right), \quad (19)$$

$$g = \frac{n^2 - 1}{4an^2} f_\omega(\vec{r}_\perp). \quad (20)$$

The last term in the right-hand side of (18) describes an external field (trap). As can be seen from (20), it corresponds to the optical waveguide. The terahertz waveguide makes an additional contribution to the condensate self-action (the second term in the right-hand side of (18)).

Let us consider a case of planar diffraction in a homogeneous medium with an anomalous group dispersion ($\Delta_\perp = 0$, $k_2 < 0$, $f_{\mathrm{T}} = f_\omega = 0$). The localized solution to Eq. (18) then has the form

$$\Phi = \left(\frac{3a}{4|b|R_\perp^2}\right)^{1/4} \exp\left(i\frac{az}{4R_\perp^2}\right)\operatorname{sech}^{1/2}\left(\frac{x}{R_\perp}\right). \quad (21)$$

Here, $R_\perp$ is the transverse size of the optical pulse.

From here and from (17), we find $R_\perp = \sqrt{\frac{1}{2}\left(\frac{\pi^2}{6} - 1\right)\frac{c}{n_{\mathrm{T}}\Omega|k_2|}}\tau_{\mathrm{p}}$,

$$\rho = \left(\frac{3a}{|b|}\right)^{1/2} \frac{1}{2R_\perp}\operatorname{sech}\left(\frac{x}{R_\perp}\right), \quad (22)$$

$$\varphi = \left[\frac{1}{12}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_{\mathrm{T}}\omega\Omega R_\perp^2} + \frac{c|k_2|\Omega^2}{n\omega}\right]z. \quad (23)$$

Since $\rho_0 = \frac{1}{2R_\perp}\sqrt{\frac{3a}{|b|}} = \frac{1}{\tau_{\mathrm{p}}}$, where τ_{p} is the duration of an optical pulse at the center of a bullet, then

$$R_\perp = \sqrt{\frac{1}{2}\left(\frac{\pi^2}{6} - 1\right)\frac{n_\perp}{c\Omega|k_2|}}R_\parallel. \quad (24)$$

Here we introduce the longitudinal size of the pulse $R_\parallel = c\tau_{\mathrm{p}}/n_{\mathrm{T}}$. We then have

$$\Omega = 0.32\frac{n_{\mathrm{T}}}{c|k_2|}\left(\frac{R_\parallel}{R_\perp}\right)^2. \quad (25)$$

The red shift of the frequency thus increases with the transverse focusing of the bullet.

Let us consider the stability of an optical-terahertz bullet. We first find the self-similar solution to system (16), (17) [13, 9–11]:

$$\rho = \frac{1}{\tau_{\mathrm{p}}}\frac{R_0}{R_\perp}F\left(\frac{x}{R_\perp}\right), \quad \varphi = f + \frac{x^2}{2}\frac{R'_\perp}{R_\perp}, \quad (26)$$

where R_0 is the transverse input aperture of the soliton.

$R_\perp$ now depends on z; according to (22), we next choose F in the form $F = \operatorname{sech}(x/R_\perp)$.

Also considering that $f_\omega = -x^2/a_\omega^2$, $f_{\mathrm{T}} = -x^2/a_{\mathrm{T}}^2$ (where a_ω and a_{T} are the characteristic scales of the transverse inhomogeneity of the waveguide in the optical and terahertz regions, respectively), after substituting (26) into (16) and (17), we find in the near-axial ($x^2/R_\perp^2 \ll 1$ [13, 11]) approximation

$$f = \frac{ck_2}{2n\omega}\Omega^2 - \left[\frac{ck_2}{2n\omega}\frac{R_0^2}{\tau_{\mathrm{p}}^2} + \frac{1}{6}\left(\frac{\pi^2}{6} - 1\right)\frac{c^2}{nn_{\mathrm{T}}\omega\Omega}\right]\frac{1}{R_\perp^2}, \quad (27)$$

$$R''_\perp = -\frac{\partial U}{\partial R_\perp}, \quad (28)$$

$$U = \frac{n^2 - 1}{2n^2 a_\omega^2}R_\perp^2 + \frac{n_{\mathrm{T}}^2 - 1}{nn_{\mathrm{T}}\omega\Omega a_{\mathrm{T}}^2}\frac{R_0^2}{\tau_{\mathrm{p}}^2}\ln\left(\frac{R_\perp}{R_0}\right) + \frac{c}{n\omega}\left[\frac{k_2R_0^2}{\tau_{\mathrm{p}}^2} + \frac{1}{2}\left(\frac{\pi^2}{6} - 1\right)\frac{c}{n_{\mathrm{T}}\Omega}\right]\frac{1}{R_\perp^2}. \quad (29)$$

Equation (28) is similar to Newton's second law for a particle of unit mass moving in an external field characterized by the potential energy $U(R_\perp)$.

Let the medium be homogeneous ($a_\omega = a_{\mathrm{T}} \to \infty$). Considering (24) too, we have $U = 0$. Hence, $R''_\perp = 0$ and $R_\perp = R(0) + R'(0)z$. Small input curvatures in the optical-pulse wavefront mean that in these sites, $R'(0) \neq 0$. Certain segments are then focused ($R'(0) < 0$), while others are defocused ($R'(0) > 0$). Due to this small-scale self-focusing, filamentation is observed. Generally speaking, the above bullet is therefore unstable. With normal group dispersion, the expression in braces in (29) is at the same time positive, which is equivalent to defocusing in a homogeneous medium. A focusing waveguide is described by the first two terms in the right-hand side of (29). It can be seen that the optical focusing waveguide forms a minimum in dependence $U(R_\perp)$. This is equivalent to the effective capture of the generated terahertz radiation. As far as the terahertz waveguide is concerned, the above theoretical consideration does not allow us to draw a similar conclusion.

NUMERICAL SIMULATION RESULTS

The theoretical analysis performed in the previous section was an averaged one and is therefore fairly rough and unable to describe very fine (small-scale and otherwise) effects. In addition, this theoretical analysis cannot describe the process of generating a terahertz pulse if it is not at the input to a nonlinear crystal.

An optical signal was supplied to the crystal during our simulation; its envelope had the form

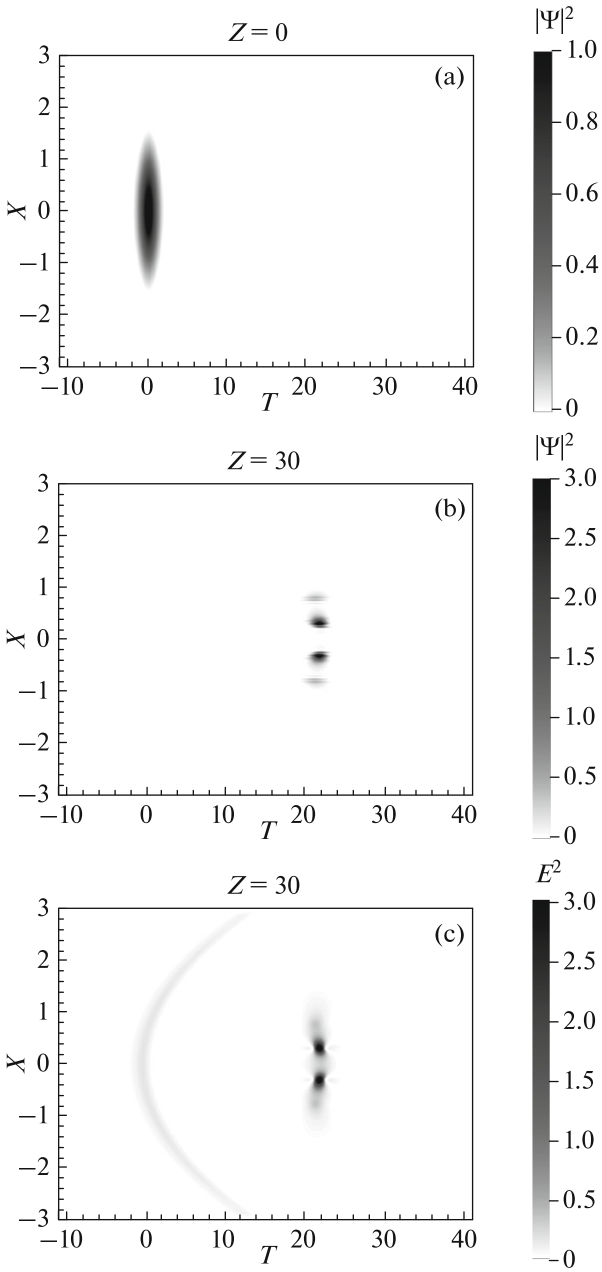

Fig. 1. Generation of a terahertz pulse in a focusing optical-terahertz waveguide with anomalous optical group dispersion. Filamentation of the optical pulse is accompanied by transverse splitting of the terahertz signal. The field amplitudes are normalized to the input peak value for the optical component.

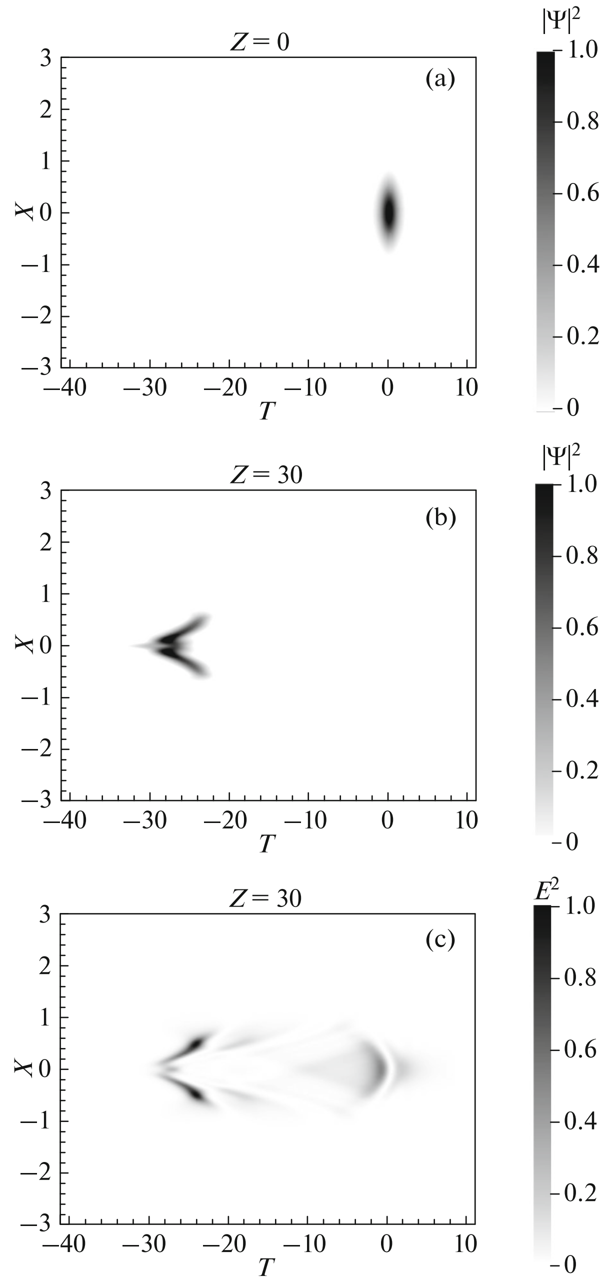

Fig. 2. Generation of a terahertz pulse in a focusing terahertz waveguide with normal optical group dispersion. The focusing of the optical component is accompanied by clearly pronounced longitudinal-transverse filamentation of the terahertz signal. The field amplitudes are normalized to the input peak value for the optical component.

$\psi = \psi_0 \,\mathrm{sech}\, T \,\mathrm{sech}\, X$, while the coordinates were nondimensionalized as $T = \tau/\tau_p$, $X = x/R_0$, $Z = z\tau_p^2/k_2$.

Figure 1 shows the generation of terahertz radiation in an optical-terahertz focusing waveguide with anomalous group dispersion in the optical region. Optical pulse filamentation is visible, and is accompanied by the clear transverse splitting of the terahertz signal into two parts. This filamentation is in agreement with the conclusion of the previous section on the instability of an optical-terahertz bullet.

Figure 2 shows the process of generation in a terahertz ($a_\omega \to \infty$) focusing waveguide with normal optical group dispersion. Filamentation of the optical pulse is in this case not as clearly pronounced. The generated terahertz signal captures it into the waveguide, in this case undergoing both transverse and longitudinal filamentation itself within the waveguide.

Numerical experiments show that with normal optical group dispersion the localization of radiation in the focusing waveguide is considerably weaker than with anomalous group dispersion. This is understood, since when $k_2 > 0$ with no waveguide, there is no localization at all.

CONCLUSIONS

Our study showed that even in the planar case, the waveguide dynamics of an optical-terahertz signal are observed in modes of small-scale filamentation. Theoretical analysis based on the averaged Lagrangian approach can therefore describe the dynamics only qualitatively. This applies especially to generation when only the optical component is present at the input into the medium. In theory, it would be better obtained with numerical experiments if we chose the trial solutions more carefully, starting with the results from numerical simulation. Analytical and numerical approaches could in this case supplement one another.

A departure from the planar approximation would be of considerable interest in further studies. It is not improbable that considering one more transverse coordinate could radically alter the waveguide dynamics of optical-terahertz pulses in the same way as in the self-focusing of optical beams in media with Kerr-type nonlinearity [14].

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