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Description of Continuum Spectrum States of Light Nuclei in the Shell Model

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Abstract—The SS-HORSE method is applied to describe the scattering of nucleons by light nuclei based on calculations within the No-Core Shell Model with JISP16 and Daejeon16 *NN* interactions. The resonant states in ^{5}He, ^{5}Li, and ^{7}He nuclei have been investigated. The SS-HORSE method generalized to the case of the democratic decay is also applied to study a four-neutron system (tetraneutron). The calculations with JISP16 and Daejeon16 and with chiral *NN* interactions indicate the existence of a rather narrow low-energy tetraneutron resonance.

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INTRODUCTION

The description of resonant states of nuclear systems including those beyond the nuclear stability line is a topical problem of nuclear physics. The investigation of such states may shed light on the nature of the internuclear forces. The so-called ab initio approaches that do not use model approximations and the entry information for which there is only the nucleon–nucleon interaction are evidently of special interest.

Currently, there are various reliable methods for the ab initio description of bound nuclear states [1], among which one can note the No-Core Shell Model (NCSM) [2]. This approach represents an advanced version of the nuclear shell model in which all nucleons are spectroscopically active and which does not contain the notion of the inert core. The nuclear wave function is expanded in a series of many-particle oscillator basis functions with the expansion including all states with the number of the oscillator excitation quanta up to a definite preset number. With the increasing number of nucleons, the basis size increases drastically and the calculation's accuracy is restricted by the computational capacities of the world's most efficient supercomputers. To date, the NCSM has been successfully applied to the description of nuclei with the number of nucleons A up to about 20.

The NCSM cannot be applied to the description of the resonant states of nuclei directly. The resonant state energies are positive relative to some decay threshold; therefore, one has to consider the possibility of the nuclear decay via different channels. To describe the resonances, special methods are required that consider the specifics of the continuum spectrum states.

Currently, reliable ab initio methods for describing the continuum spectrum states based on the Faddeev and Faddeev–Yakubovsky equations are successfully applied in nuclear physics to systems with $A \leq 5$ (see, e.g., [1, 3]). There are NCSM generalizations that use the resonating-group method [4], which were used for calculating individual nuclear systems that contained up to 11 nucleons [5]. These methods are, however, rather complicated from the point of view of the numerical implementation and require considerable additional computational resources.

Recently, the authors have proposed the Single-State Harmonic Oscillator Representation of Scattering Equations (SS-HORSE) method [6–10], a generalization of the NCSM based on the harmonic oscillator representation of scattering equations (HORSE) [11] for continuum spectrum states. The SS-HORSE method allows calculating the S-matrix and resonant parameters of the single-channel scattering based on the NCSM eigenenergies above the breakup threshold.

This article provides a brief review of the SS-HORSE method (Section 1) and results for the single-channel $n\alpha$ scattering and resonances in the ^{5}He nucleus (Section 2) and for the $p\alpha$ scattering and resonances in the ^{5}Li nucleus (Section 3). Then, new results of analysis of the neutron scattering by the ^{6}He nucleus and resonances in the ^{7}He nucleus are presented (Section 4). In conclusion, results of a search for the resonance in a four-neutron system (Section 5) obtained in the NCSM with different *NN* forces are collated with calculated results with the JISP16 interaction published earlier [12] and experimental results currently available [13] are presented.

1. THE SS-HORSE METHOD

Let us briefly consider the SS-HORSE method exemplified by the problem of the scattering of a nucleon by a nucleus with mass number A.

Let us denote by $E_\nu^{A+1}(\hbar\Omega, N_{\max})$ the energy of a definite state ν of the continuum spectrum of a system with $A + 1$ nucleons calculated in the NCSM model space that considers all many-particle states with the total number of the oscillator excitation quanta up to $N_{\max}$ and with parameter of the NCSM oscillator basis $\hbar\Omega$ and by $E_0^A\left(\hbar\Omega, N'_{\max}\right)$, the energy of the ground state of target A obtained in the NCSM with the same value of parameter $\hbar\Omega$ and the total number of the oscillator excitation quanta $N'_{\max} = N_{\max}$ or $N_{\max} - 1$ depending on the parity of state ν.

The phase shifts in the SS-HORSE in the absence of the Coulomb interaction are calculated by the formula [6, 7, 10]

$$\tan\delta_\ell(E_\nu) = -\frac{S_{\mathbb{N}+2,\ell}(E_\nu)}{C_{\mathbb{N}+2,\ell}(E_\nu)}. \quad (1)$$

Here, $S_{n,\ell}(E)$ and $C_{n,\ell}(E)$ are regular and irregular oscillator solutions for a free Hamiltonian with the solutions known in explicit forms [11], where ℓ is the orbital moment of the relative motion, and $\mathbb{N}$ is the total number of the oscillator quanta of a system of $A + 1$ particles, which is convenient to represent in the form of the sum of oscillator excitation quanta $N_{\max}$ that sets the sizes of the model space, and the minimum possible number of the oscillator quanta of the system $N_{\min}$: $\mathbb{N} = N_{\max} + N_{\min}$. The target is assumed to be in the ground state, i.e., all excitation quanta fall in the energy of the relative motion as

$$E_\nu = E_\nu^{A+1}(\hbar\Omega, N_{\max}) - E_0^A\left(\hbar\Omega, N'_{\max}\right), \quad (2)$$

counted from the reaction threshold.

In the case of scattering of charged particles, the phase shifts based on the NCSM eigenenergies can be calculated by the formula [9–11]

$$\tan\delta_\ell(E_\nu) = -\frac{S_{\mathbb{N}+2,\ell}(E_\nu)W_b(n_\ell, F_\ell) + C_{\mathbb{N}+2,\ell}(E_\nu)W_b(j_\ell, F_\ell)}{S_{\mathbb{N}+2,\ell}(E_\nu)W_b(n_\ell, G_\ell) + C_{\mathbb{N}+2,\ell}(E_\nu)W_b(j_\ell, G_\ell)}. \quad (3)$$

Here, $j_l \equiv j_l(kr)$ and $n_l \equiv n_l(kr)$ are the spherical Bessel and Neumann functions, respectively, [14]; $F_l \equiv F_l(\eta, kr)$ and $G_l \equiv G_l(\eta, kr)$ are the regular and irregular Coulomb functions, respectively, [14]; $\eta = Z_1Z_2e^2\mu/(\hbar^2k)$ is the Sommerfeld parameter, Z_1e and Z_2e are the charges of the scattering particles, μ is the reduced mass of the latter, and k is the momentum of the relative motion. The quasi-Wronskian $W_b(\phi, \chi)$ is defined as

$$W_b(\phi,\chi) = \left.\left(\frac{d\phi}{dr}\chi - \phi\frac{d\chi}{dr}\right)\right|_{r=b}. \quad (4)$$

As shown in [9], the optimal value of parameter b for calculation of the quasi-Wronskian $W_b(\phi, \chi)$ is the so-called [11] natural channel radius $b = \sqrt{\hbar(2\mathbb{N}+7)/(\mu\Omega)}$.

Consequently, in the SS-HORSE method, the phase shift at the eigenenergy of the Hamiltonian E_ν is determined only by the parameters of the NCSM oscillator basis $N_{\max}$ and $\hbar\Omega$, which are contained in the definitions of functions $S_{n,\ell}(E)$ and $C_{n,\ell}(E)$ in Eq. (1) and by varying these parameters one can calculate phase shifts $\delta_\ell(E_\nu)$ in a definite energy range. Then, using an appropriate parameterization, one can obtain smooth dependence of $\delta_\ell(E_\nu)$ in the energy interval of interest and calculate the energies and widths of the resonances. An important requirement on the parameterization is ensuring the low-energy behavior of the phase shifts correct from the point of view of the quantum theory of scattering.

We considered different options for the parameterization of the phase shifts. The parameterization based on the symmetry properties of the S-matrix [6] is illustrative; in this case, the adjustable parameters include the energy and width of the resonance under study. The merit of the other variant based on analytic properties of the effective radius function [10] is the possibility of investigating the scattering of neutral and charged particles from a common standpoint. The number of the adjustable parameters is in this case smaller; however, the characteristics of the resonance are should be found in an additional calculation based on the parameterization performed.

2. THE $n\alpha$ SCATTERING AND THE RESONANCES IN THE ^{5}He NUCLEUS

To describe the $n\alpha$ scattering, in the NCSM the energies of the lowest states of the ^{5}He nucleus with $J^\pi = \frac{3}{2}^-, \frac{1}{2}^-, \frac{1}{2}^+$, and the ground state of the ^{4}He nucleus in the bases with $N_{\max} \le 18$ and the values of $\hbar\Omega$ from the interval $10 \le \hbar\Omega \le 40$ MeV were calculated. We used the JISP16 [15] and Daejeon16 interactions [16] as nucleon–nucleon forces.

Let us illustrate how the method works by the example of the $n\alpha$ scattering in the $\frac{3}{2}^-$ state. In the left panel of Fig. 1, the symbols show the energies of the relative motion of $n\alpha$ calculated according to Eq. (2) based on the results of the NCSM with the JISP16

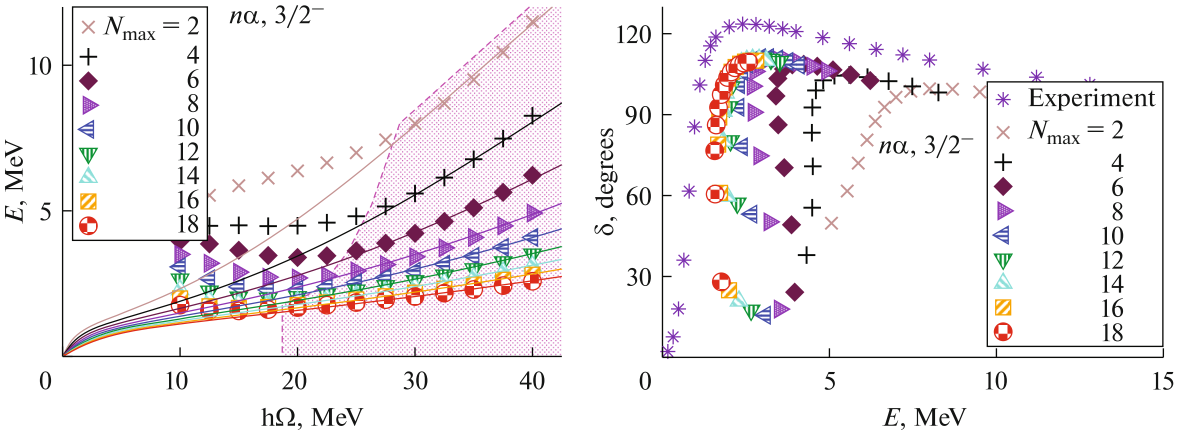

Fig. 1. Left panel: the symbols denote the eigenenergies of the relative motion of $n\alpha$ obtained in the NCSM with the JISP16 NN interaction for the $\frac{3}{2}^{-}$ state; in the shaded region, the values selected for the parameterization of the phase shifts in the SS-HORSE method are shown; the solid curves were constructed based on the above parameterization. Right panel: the symbols denote the phase shifts calculated by Eq. (1); the asterisks denote the experimental data of [17].

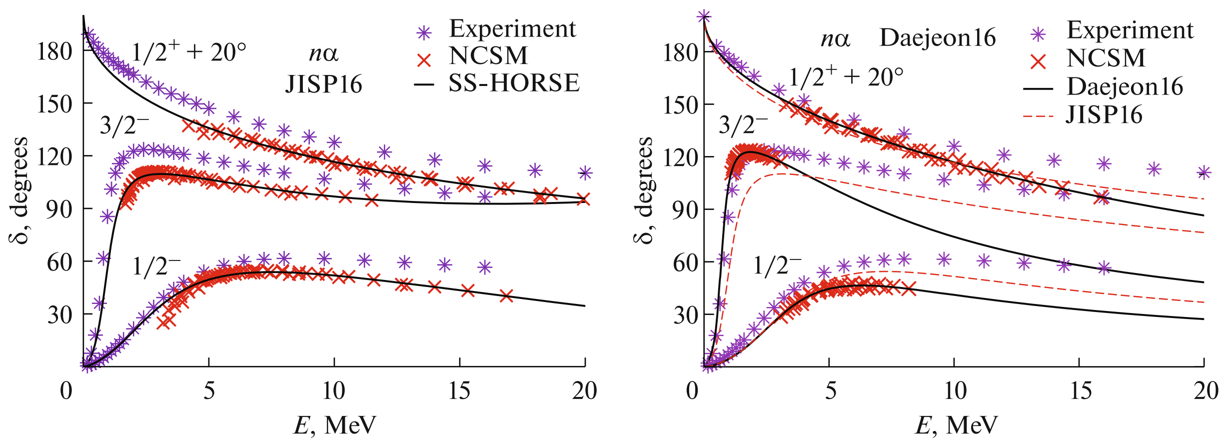

Fig. 2. The phase shifts for different $n\alpha$ scattering states obtained in the SS-HORSE method based on the NCSM calculations with the JISP16 (left panel) and Daejeon16 (right panel) NN interactions. The crosses denote the results obtained by using Eq. (1) with the selected data, and the asterisks denote the experimental scattering phase shifts of [17]. The solid curves represent the parameterization results. The experimental data and the theoretical results for the nonresonant $\frac{1}{2}^{+}$ scattering are shifted upwards by 20°. The dashed curves are provided in the right panel for comparison of the phase parameterization obtained with the JISP16 interaction.

interaction for the ^{5}He and ^{4}He nuclei. In the right panel of Fig. 1, the corresponding symbols show the phase shifts obtained according to Eq. (1). With the increasing value of N_{max}, increasingly more points fall on one smooth curve. The results obtained with the Daejeon16 interaction behave in the same way. To parameterize the scattering phase shifts, only the results were used that form this smooth dependence. The details of the selection of the data and execution of the parameterization itself are thoroughly considered in [6, 10]. Here, we provide the final results: the selected energies are located in the shaded region in the left panel of Fig. 1 and the respective phase shifts that form the smooth curve are shown by crosses in Fig. 2.

The parameterization results are represented by the curves of the phase shifts in Fig. 2. The curves of dependences $E_0(\hbar\Omega)$ in different model spaces in the left panel of Fig. 1 correspond to the parameterized phase for the JISP16 NN interactions. It can be seen that these curves reproduce well the energies from the shaded region obtained in the NCSM and selected for the parameterization of the phase shifts.

Table 1. Resonance parameters obtained in the SS-HORSE method for the ^{5}He, ^{5}Li, and ^{7}He nuclei with the Daejeon16 and JISP16 *NN*-interactions. For comparison, experimental results for ^{5}He and ^{5}Li from [18] and ^{7}He from [19] are provided

Nucleus	State		Daejeon16	JISP16	Experiment
^{5}He	$\frac{3}{2}^-$	E_r	0.68	0.89	0.80
		Γ	0.52	0.99	0.65
	$\frac{1}{2}^-$	E_r	2.45	1.86	2.07
		Γ	5.07	5.46	5.57
^{5}Li	$\frac{3}{2}^-$	E_r	1.52	1.84	1.69
		Γ	1.05	1.80	1.23
	$\frac{1}{2}^-$	E_r	3.21	3.54	3.18
		Γ	5.63	6.04	6.60
^{7}He	$\frac{3}{2}^-$	E_r	0.27	0.71	0.44
		Γ	0.12	0.61	0.15
	$\frac{1}{2}^-$	E_r	2.7	2.8	1.2
		Γ	4.2	5.01	1.0
	$\frac{5}{2}^-$	E_r	3.65	4.37	3.36
		Γ	1.37	1.55	1.99

The scattering phase shifts in other states were obtained in a similar way. The results of analysis of the $n\alpha$ scattering phase shifts in the $\frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{1}{2}^+$ states according to the selected data that form the smooth dependence of a corresponding phase shift on the energy are indicated in Fig. 2 by crosses. The curves constructed according to the parameterization results are in reasonable agreement with the phase shift analysis of the experimental data of [17]. We should note that in the resonant state $\frac{3}{2}^-$, the phase shifts obtained with Daejeon16 are noticeably closer to the experiment than the phase shifts obtained with JISP16, although in the resonant state $\frac{1}{2}^-$ and in the nonresonant scattering $\frac{1}{2}^+$, the phase shifts obtained with these interactions are close to each other. We should stress that the SS-HORSE method allows a good description of not only the resonant but also nonresonant $n\alpha$ scattering in the $\frac{1}{2}^+$ state, although the low-lying continuum spectrum states obtained in the nuclear shell model are traditionally associated only with the resonant states.

The energies and widths of the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ resonant states of the ^{5}He nucleus found in the SS-HORSE method based on the NCSM calculations (see Table 1) are also in good agreement with experimental data [18] with the parameters of the $\frac{3}{2}^-$ resonance being described by the Daejeon16 interaction somewhat better while those of the $\frac{1}{2}^-$ resonance being described better by the JISP16 interaction.

3. THE $p\alpha$ SCATTERING AND THE RESONANCES IN THE ^{5}Li NUCLEUS

To describe the phase shifts of the $p\alpha$ scattering and the resonances of the ^{5}Li nucleus by the SS-HORSE method, in the NCSM the energies of the lowest states with $J^\pi = \frac{3}{2}^-, \frac{1}{2}^-$, and $\frac{1}{2}^+$ of the ^{5}Li nucleus were calculated. Like the case of the $n\alpha$ scattering, the calculations were performed in the NCSM with two *NN* interaction variants, namely, JISP16 and Daejeon16.

In Fig. 3, the crosses correspond to the calculations of the phase shifts by Eq. (3) for the selected energy values of the states listed above. The curves in the graphs represent the results of the phase shift parameterization based on a modified effective radius function [10]. Using this parameterization, the *S*-matrix was calculated and the positions of its poles were found numerically, which allowed the determination of the energies and widths of the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ resonances of the ^{5}Li nucleus presented in Table 1. The Daejeon16 interaction describes on the whole both the phase shifts and resonances somewhat more accurately than the JISP16 interaction.

4. THE n–^{6}He SCATTERING AND THE RESONANCES IN THE ^{7}He NUCLEUS

To describe the n–^{6}He scattering and the resonances in the ^{7}He, in the NCSM the energies of the lowest states with $J^\pi = \frac{3}{2}^-, \frac{1}{2}^-, \frac{5}{2}^-$, and $\frac{1}{2}^+$ of the ^{7}He nucleus and the ground state of the ^{6}He nucleus were calculated. The calculations were conducted in the NCSM with the JISP16 and Daejeon16 *NN* interactions in the bases with $N_{\max} \le 17$ and the values of $\hbar\Omega$ from the interval $10 \le \hbar\Omega \le 50$ MeV. For the $\frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{5}{2}^-$ resonant states and the nonresonant $\frac{1}{2}^+$ scattering of the neutron by the ^{6}He nucleus, the convergence of the SS-HORSE results for the scattering phase shifts was achieved in total with the convergence of the

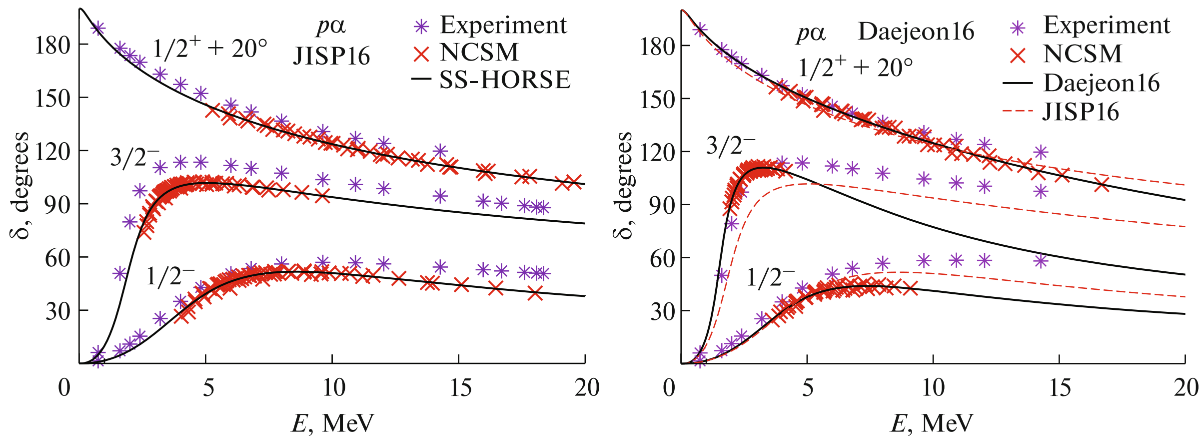

Fig. 3. The phase shifts for different $p\alpha$ scattering states obtained in the SS-HORSE method based on the NCSM calculations with the JISP16 (left panel) and Daejeon16 (right panel) *NN* interactions. The crosses denote the results calculated by Eq. (3) with the selected data and the asterisks denote the experimental scattering phase shifts of [20]. The experimental data and the theoretical results for the nonresonant $\frac{1}{2}^+$ scattering are shifted upwards by 20°. For the rest of the notation, refer to Fig. 2.

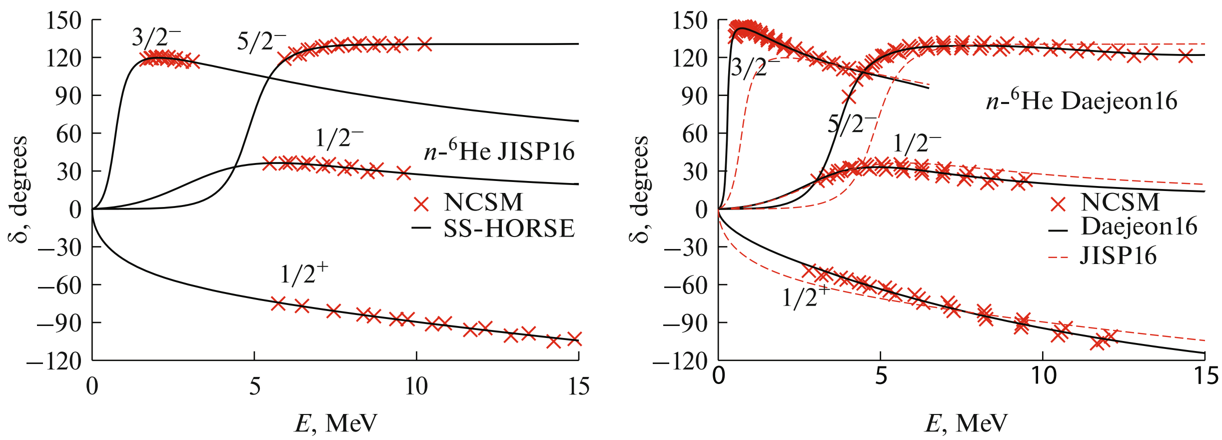

Fig. 4. The phase shifts for different n–^{6}He scattering states obtained in the SS-HORSE method based on the NCSM calculations with the JISP16 (left panel) and Daejeon16 (right panel) *NN* interactions. For the notation, refer to Fig. 2.

results calculated with the Daejeon16 interaction being better than that obtained with JISP16.

In Fig. 4, the phase shifts obtained in the SS-HORSE method are shown. For comparison of the results obtained with different interactions, both the phase shifts resulting from the Daejeon16 and those obtained using the JISP16 are shown in the right-hand panel of Fig. 4.

The energies and widths of the resonances of the ^{7}He nucleus presented in Table 1 were found by numerically determining the positions of the *S*-matrix poles using its parameterization that corresponds to the parameterization of the scattering phase shifts. The parameters of the resonances obtained in the SS-HORSE method are close to the experimental parameters of [19]. On the whole, the Daejeon16 yields resonances with a somewhat lower energy than those yielded by the JISP16, and hence the Daejeon16 resonances are in better agreement with the experiment.

5. THE RESONANCES IN A FOUR-NEUTRON SYSTEM

To search for the resonances in an exotic nucleon system consisting of four neutrons (tetraneutron), the SS-HORSE method was generalized [12] to the case of true many-particle scattering [21, 22] using the formalism of [23]. In the simplest approximation, the wave function of a tetraneutron is described by only one hyperspherical harmonic with the lowest possible value of the hypermoment $K = 2$. In such a single-channel case, the *S*-matrix of a many-particle system

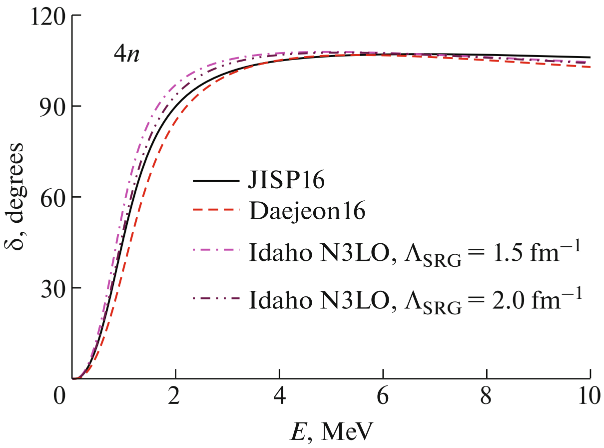

Fig. 5. Phase shift δ of the 4 → 4 scattering for a four-neutron system obtained in the SS-HORSE method using various *NN* interactions.

can be expressed by phase shift δ of the 4 → 4 scattering as

$$S = e^{2i\delta}. \quad (5)$$

This phase shift can be calculated in the SS-HORSE at eigenenergies E_ν of the tetraneutron Hamiltonian obtained in the NCSM as

$$\tan\delta(E_\nu) = -\frac{S_{N_{\max}+4,\mathcal{L}}(E_\nu)}{C_{N_{\max}+4,\mathcal{L}}(E_\nu)}. \quad (6)$$

Here, $S_{n,\ell}(E)$ and $C_{n,\ell}(E)$ are the functions introduced in [23], which coincide in the case under consideration with the functions used in Eq. (1); the minimum number of the oscillator quanta in the four-neutron system equals two, therefore, $\mathbb{N} = N_{\max} + 2$; and the effective angular momentum $\mathcal{L} = K + 3 = 5$ given that we use the only hyperspherical harmonic with the hypermoment $K = 2$.

To search for the resonances, the ground-state energies of the four-neutron system were calculated in the NCSM in the bases with $N_{\max} \leq 20$ and the values of $\hbar\Omega$ from the interval $1 \leq \hbar\Omega \leq 50$ MeV with different *NN* forces, namely, JISP16, Daejeon16, and Idaho N3LO potential [24] obtained in the chiral effective field theory and "mitigated" by the SRG transformation [25, 26] with the parameters $\Lambda = 1.5$ and $2.0\ \mathrm{fm}^{-1}$.

Table 2. Energies E_r and widths Γ of the resonance of the tetraneutron and energies E_f of the false pole of the *S*-matrix obtained in the SS-HORSE method using different *NN* interactions

	JISP16	Daejeon16	Idaho N3LO, SRG	
			$\Lambda = 1.5\ \mathrm{fm}^{-1}$	$\Lambda = 2.0\ \mathrm{fm}^{-1}$
E_r, MeV	0.844	0.997	0.783	0.846
Γ, MeV	1.38	1.60	1.15	1.29
E_f, keV	–54.9	–63.4	–52.1	–54.5

The results of the parameterization of phase shifts δ calculated by Eq. (6) are shown in Fig. 5—the parameterization procedure is completely similar to that considered in [12]. All *NN*-interaction models considered yield similar dependences of phase shifts δ on the energy, which suggests the presence of a rather narrow resonance. Furthermore, the SS-HORSE method indicates the presence of not only a resonant but also a false pole of the *S*-matrix in the four-neutron system, the parameters of which are presented in Table 2. The positions of the poles do not change greatly depending on the interaction and are in good agreement with the results of a recent experiment [13] that showed that the width of the tetraneutron's resonance did not exceed 2 MeV and the resonance energy is $E_r = 0.83 \pm 0.65$ (stat.) ± 1.25 (syst.) MeV.

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