‘Training’ of photorefractive self-pumped phase-conjugate mirrors

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ABSTRACT

We show the speed of response of a photorefractive self-pumped phase-conjugate mirror can be considerably (6 - 20 times) accelerated by special training procedures. Efficiency of some such procedures is compared and analyzed

Keywords: self-pumped phase-conjugate mirror, photorefractive nonlinearity, speed of nonlinear response

1. INTRODUCTION

Self-pumped phase-conjugate (SPPC) mirrors based on photorefractive crystals (PRCs) [1] can operate with CW radiation intensities down to a few mW/cm^2 [2] and do not even require an additional pumping [3,4], being threshold-free (with respect to radiation intensity but not to nonlinear constant) optical parametric oscillators. For example, in so-called double SPPC mirrors [5-7], two incoherent [8], orthogonally polarized [9] or frequency-not-coincident [10] light waves can be simultaneously conjugated. In this case, generation is developed due to self-organization (a phase transition) under irradiation [11,12] what looks similar to formation of PC wave in SRS and SBS SPPC mirrors [13]. Therefore, while steady-state nonlinear reflection may be as high as 0.8-0.9 and more, the transient time of such a mirror is rather long and may exceed tens of seconds [14]. Note also that well-known techniques based on varying the problem geometry [14], increasing the seed noise [13,15], using the frequency shifts [16,17] and external DC [18,19] or AC [20-22] electric fields can not be utilized here. They either sharply reduce the mirror efficiency or cause development of instabilities (rather complicated self-oscillations with characteristic times up to a few hours and days [2]).

Below, by the example of a loop SPPC mirror based on a $BaTiO_3$ crystal and using the calculation procedures described in [23,24], we show that PC wave formation can be considerably fastened by a special preliminary ‘training’ procedure that is by the use of an additional (training) optical field to write preliminary dynamic holograms in PRC. These holograms provide later a much more rapid formation of refractive-index gratings required for efficient conjugation of an input signal. Obviously, such training can not be universal and to realize it one needs to know something about the input signal properties.

The concept of such a training is close to pattern recognition problems [25,26], development of optical correlators [27] and elements of associative memory [28-30]. In [30], a possibility to write simultaneously some time-separated holographic images in PRC was studied. At first (‘training’) stage, two different (‘positive’ and ‘negative’) optical images with full contrast inversion, produced by a liquid-crystal transparency, were interchanged on a SPPC mirror input at the frequency $f_m = 30$ frames/second. Period $\tau_m = l/f_m$ was much shorter than characteristic times of PRC. After the end of this stage transient, the second (‘conjugation’) stage was started. During this stage, new input images, formed by the same transparency, were projected onto the mirror. Two most interesting results were here obtained. First, it was found that after the end of the first stage transient, the output field spatial structure does never coincide with both input images. Second, it was shown that, at the second stage, the time needed to obtain a steady-state nonlinear response drastically depends on correlation of a new input image with the images used at the training stage.

2. THE MODEL

Figure la illustrates geometry of considered model problem. As in [23], we assumed that forward and backward light waves with amplitudes $A_{f,b}$ and wave vectors $\mathbf{k}_{f,b} = \{k_x, \pm k_z\}$ propagate from planes $z=0$ and $z=L$ at a small angle $\alpha/2$ ($k_z \gg k_x$) to positive (negative) direction of z axis, respectively. It was assumed the wave A_b is formed by a special optical system consisting of two fold mirrors and a lens. The mirrors change the propagation direction of the wave A_f by the angle $(\pi-\alpha)$, while the lens projects without scaling the field distribution. This provides the condition

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$A_b(x,z=L,t) = A_b(-x,z=L,t)\exp(-ikx\sin\alpha)$. As in [23,24], nonlinear response kinetics was calculated by the use of the same microscopic equations [31] written for the two-dimensional case [32] with taking into account only transmission dynamic holograms [vector $\boldsymbol{\kappa}$ of refractive-index gratings $\delta\eta(x,z,t)$ is directed along $\mathbf{x}$ axis] by neglecting the photovoltaic effect [1,14,18]. It was assumed that external DC field $\mathbf{E}_0$ directed along $\mathbf{x}$ axis is rather weak ($E_0 = 1$ V/cm). The problem was considered to be self-consistent with taking into account relation of intensity distributions $I_{f,b}(x,z,t) = |A_{f,b}(x,z,t)|^2$ with internal electric field $E_{sc}(x,t) \propto \delta\eta(x,z,t)$. Truncated wave equations for the amplitudes $A_{f,b}(x,z,t)$ of interacting waves were written in paraxial approximation. As in [23,24], we considered the case when total intensity was determined by the sum of intensities $I(x,z,t) = I_f(x,z,t) + I_b(x,z,t)$, i.e., the case of incoherent or orthogonally polarized counter-propagating waves.

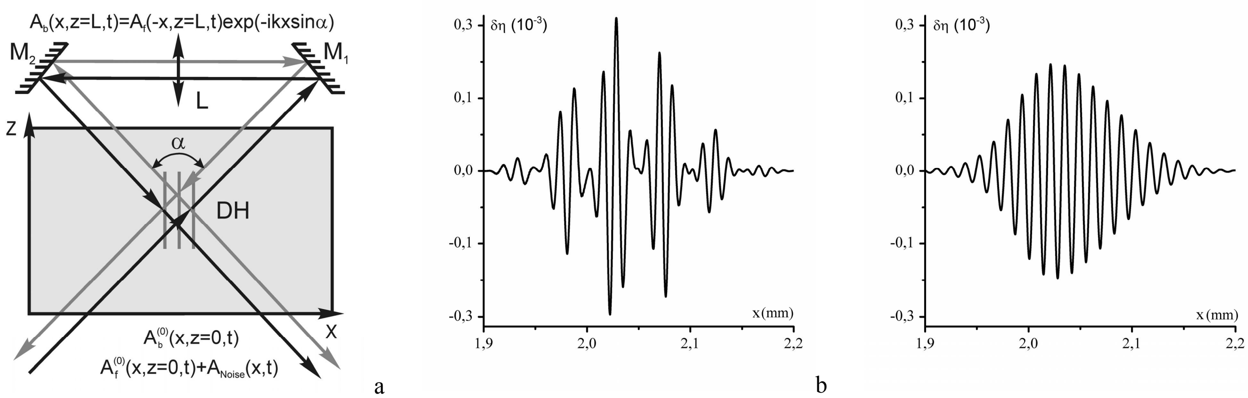


Fig.1. Interaction geometry (a) and spatial distribution $\delta\eta(x)$ (b) in plane $z = L/2$ for the case of steady-state conjugation of input Gaussian beam with (a) and without (b) spatial modulation ($\Lambda_m = 100$ μm) of $A_f(x, z = 0)$. The beam diameter is $2\rho_0 = 230$ μm, α = 14°, $\langle I_{Noise}\rangle / I_{max} = 10^{-4}$, $I_{max} = 55$ (b) and 35 (c) mW/cm^2, $E_0 = 1$ V/cm.

The self-consistent problem was solved numerically by calculating the evolution of $A_{f,b}(x,z,t)$ and $\delta\eta(x,z,t)$ in time. All variables were described on the grid with full number of nodes equal to 8192 (along PRC width h = 4 mm) and 512 (along PRC length l = 4 mm). The initial conditions corresponded to 'switching on' the mirror at t = 0. After that ($t \geq 0$), the input field $A_f(x,z=0,t)$ was defined as a superposition of the 'useful' (training or information) signal $A_f^{(0)}(x,z=0,t)$ and delta-correlated noise $A_{Noise}(x,t)$ with average intensity $\langle I_{Noise}\rangle = \langle |A_{Noise}(x,t)|^2 \rangle$ equal to 10^{-4} of the useful signal maximal intensity $I_{max} = 35$ or 55 mW/cm^2. The problem was solved within the limits of adiabatic approximation by using the method of separation over physical factors and the fast Fourier transform [33,34]. The step in time ($\Delta t \approx 0{,}15$ s) was much smaller than evolution time of PRC state. Spatial period of dynamic holograms was specified by the convergence angle α = 14°. Most other parameters were not varied and their values were determined by the PRC type ($BaTiO_3$, see Table 1 in [23,24]). Spatial distribution of input radiation was defined as

$$A_f^{(0)}(x, z = 0, t) = G(x)M(x, t), \tag{1}$$

where $G(x)$ and $M(x,t)$ describe Gaussian envelope of the input beam with diameter $2\rho_0 = 230$ μm and its spatial (information) modulation. It is the instantaneous change in $M(x,t)$ at moments $t = 0$ corresponding to the beginning [$M(x, t < 0) \equiv 0 \Rightarrow M(x, t \geq 0) \neq 0$] and the end [$M(x, t < 0) \neq 0 \Rightarrow M(x, t \geq 0) \neq 0$] of the mirror training stage (the time reading was started at these moments anew) that simulated the transient process whose acceleration is the main goal of this paper. The form of the function $M(x,t)$ was different for different realizations and will be described below.

Because we optimized earlier [23] all the parameters of the problem, SPPC mirror conjugated the input beam both in the absence of its information modulation [$M(x,t\ge 0)=1$] and in its presence [$M(x,t\ge 0)=\sin(2\pi\kappa_m x)$, $\Lambda_m=\kappa_m^{-1}=100$ μm]. Figures lb,c show central ($z=L/2$) cross-section of steady-state (t = 150 s) dynamic holograms [distributions $\delta\eta(x,z)$] in both these cases.

Kinetics of the product RH of the nonlinear reflection coefficient

$$R(t)=\int_0^{H/2}\left|A_b(x,z=0,t)\right|^2 dx\bigg/\int_0^{H/2}\left|A_f(x,z=0,t)\right|^2 dx \qquad (2)$$

and the overlapping integral

$$H(t)=\left|\int_0^{H/2}A_f(x,z=0,t)A_b^*(x,z=0,t)\,dx\right|^2\bigg/\left[\int_0^{H/2}\left|A_f(x,z=0,t)\right|^2 dx\int_0^{H/2}\left|A_b(x,z=0,t)\right|^2 dx\right], \qquad (3)$$

which obviously describes efficiency of phase conjugation, is illustrated for both these cases in Figure 2. One can easy to check the transient time proves to be very long. Defining it as the time τ_t required for the parameter RH to achieve 90% of its maximal value $(RH)_{max}$, we obtain $\tau_t=60$ and 65 s in the absence (Figure 2a) and presence (Figure 2b) of harmonic information modulation.

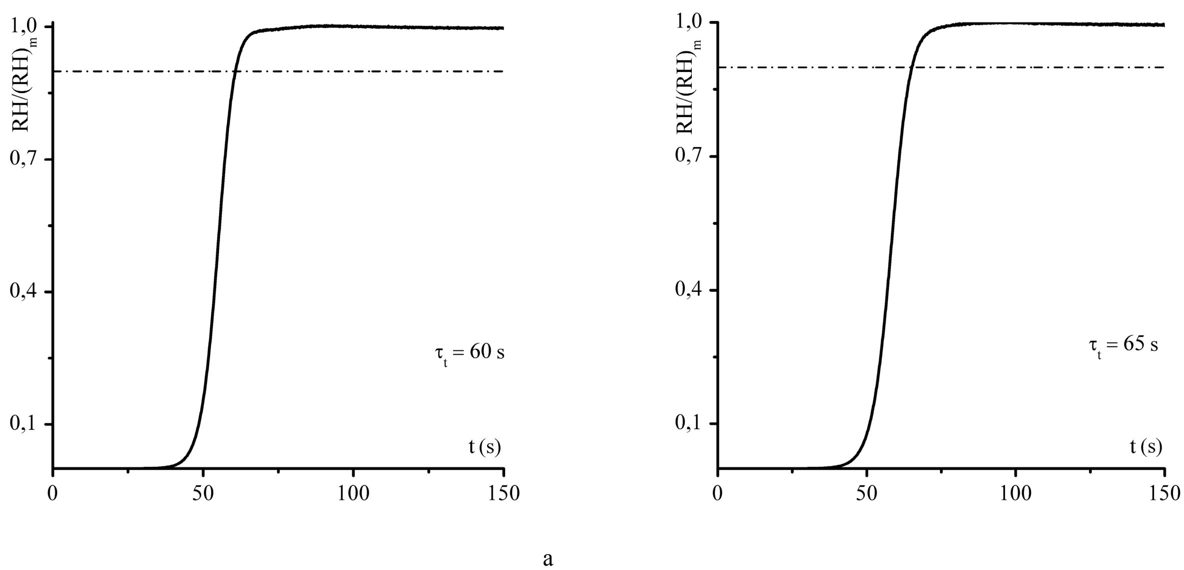
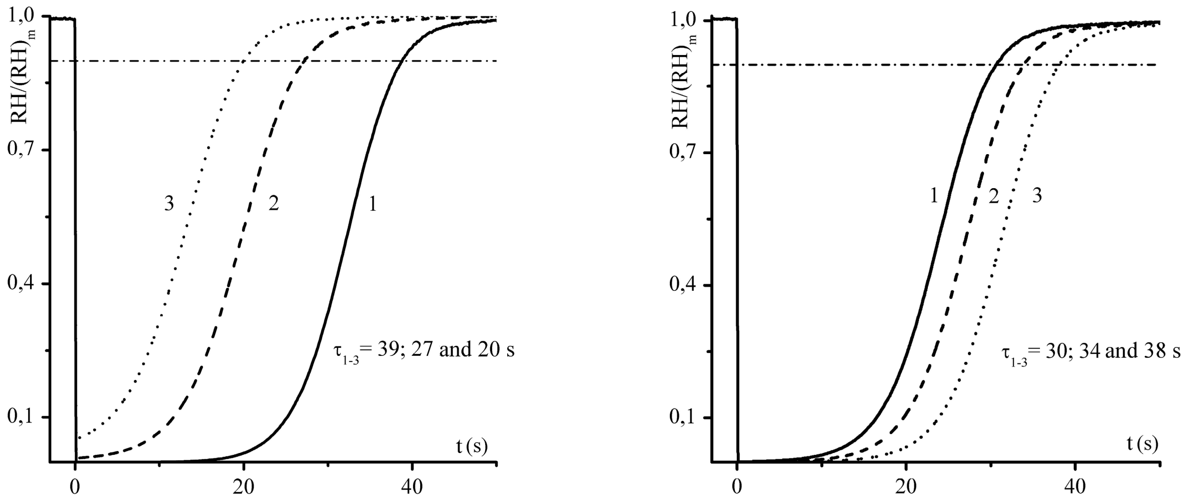


Fig.2. Transient processes for RH upon conjugation of a Gaussian beam without spatial harmonic modulation $M(x)$ of $A_f(x,z=0)$ ($I_{max}=35$ mW/cm^2, a) and upon modulation with the period $\Lambda_m=100$ μm ($I_{max}=55$ mW/cm^2, b).

3. STATIC TRAINING

A training can not be universal because the choice of training field optimal spatial structure requires consideration of a number of properties of information signal which wave front will be conjugated. Therefore, we assume below that almost all characteristics of this signal are known. We suppose that after the start this signal looks like described above Gaussian beam with harmonically modulated amplitude. However, the phase φ and spatial period $\Lambda_m=\kappa_m^{-1}$ of the function $M(x,t\ge 0)=\sin(2\pi\kappa_m x+\varphi)$ (where t = 0 is the start moment) in (1) are unknown. Note that all presented results will be related to three possible values of $\Lambda_m=100$, 90, and 80 μm and the worst case of φ choice after the start.

From this point of view, steady-state distribution $\delta\eta(x,z)$ shown in Figure Ib can be considered as a new initial state (with respect to the start moment t = 0 of phase conjugation) of the mirror obtained due to its training by a Gaussian beam with harmonically modulated amplitude. Because such training neglects a possibility of different phase φ of the information signal, it was unlikely that this procedure might be efficient. This was confirmed by our simulation. In case when the phase φ of $M(x)$ shifts jump-wise by $\pi/2$ at the moment t = 0 and Λ_m simultaneously changes [$\Lambda_m = 100$, 90, and 80 μm for curves (1), (2), and (3)], RH kinetics is shown in Figure 3a. It is easy to see that while the transient time becomes a bit shorter ($\tau_{1-3} = 39$, 27, and 20 s for $\Lambda_m = 100$, 90, and 80 μm), this advantage is not so large and, what is much more important, noticeably decreases when Λ_m for training and conjugated signals coincide. This clearly indicates that there is no sense to use this procedure to write initial information on the expected value of Λ_m.



a b

Fig.3. SPPC mirror training by a spatially modulated beam ($\Lambda_m = 100$ μm, a) and a beam without modulation (b). Transient processes for RH at the training stage (a) and after the start (b). During the start (the moment t = 0), the phase of $M(x)$ shifts by $\pi/2$ and Λ_m changes [$\Lambda_m = 100$, 90, and 80 μm for curves (1), (2), and (3)].

Taking this into account, one can expect that elimination of Λ_m initial information during the training stage may play a positive role. This was confirmed by our calculations. Figure 3b [curves (1), (2), and (3)] illustrates kinetics of RH in case of a beam with smooth Gaussian envelope for which I_{max} value changes jump-wise from 35 to 55 mW/cm^2 at the moment t = 0 (that is total beam power being preserved) and harmonic modulation $M(x)$ with spatial period $\Lambda_m = 100$, 90, and 80 μm simultaneously appears. One can see that in this case a character of dependence $\tau(\Lambda_m)$ becomes opposite ($\tau_{1-3} = 30$, 34, and 38 s for $\Lambda_m = 100$, 90, and 80 μm). However, the advantage obtained here due to the training proves to be also too small.

4. DYNAMIC TRAINING

Because during the mirror training we should write in PRC the information on Λ_m expected value, the mirror should be irradiated with the spatial modulation frequency corresponding to Λ_m. At the same time, because information on the phase φ of signal spatial modulation is absent, it is impossible to memorize $M(x)$ during the training. Taking into account that PRC is very inertial, these two requirements can be simply satisfied by a rather fast change of φ in time. Such a situation can be realized, for example, in case when the input signal is modulated by the function $M(x,t) = \sin\left[2\pi\left(\kappa_m x + f_m t\right)\right]$, where the modulation frequency f_m is so high that refractive-index gratings $\delta\eta(x,z)$ have no time to follow its variations. Figure 4 illustrates results of such training. As follows from Figure 4a, the training stage ($\Lambda_m = 100$ μm, the phase φ changes by 2π for 2 s) proves to be rather long (almost 200 s) and what is very interesting even after that RH value oscillates at the frequency f_m of external 'force' $M(x,t)$ (see inset in Figure 4a).

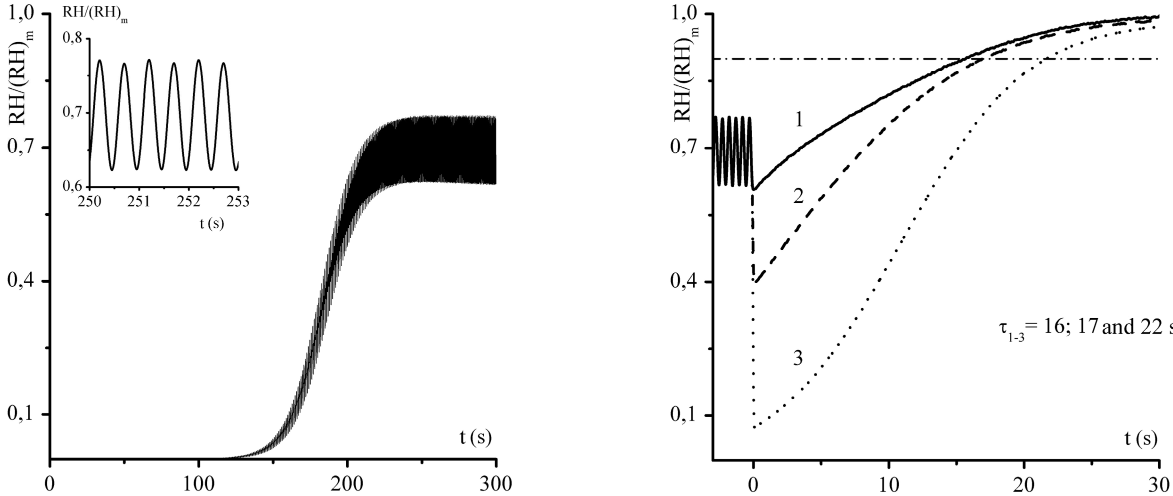


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Fig.4. SPPC mirror training by a beam with the phase of $M(x)$ linearly increasing in time ($\Lambda_m = 100$ μm, the phase changes by 2π for 2 s). Transient processes for RH at the training stage (a) and after the start (b). During the start, the phase of $M(x)$ shifts by $\pi/2$ and Λ_m changes [$\Lambda_m = 100$, 90, and 80 μm for curves (1), (2), and (3)].

After such training, kinetics of RH for Gaussian beam with jumps in φ (by $\pi/2$) and Λ_m [$\Lambda_m = 100$, 90, and 80 μm for curves (1), (2), and (3)] at the start moment (t = 0) is illustrated in Figure 4b. One can see that RH transient takes now considerably shorter time ($\tau_{1-3} = 16$, 17, and 22 s for $\Lambda_m = 100$, 90, and 80 μm) and the advantage in τ achieved due to the mirror preliminary training is much greater.

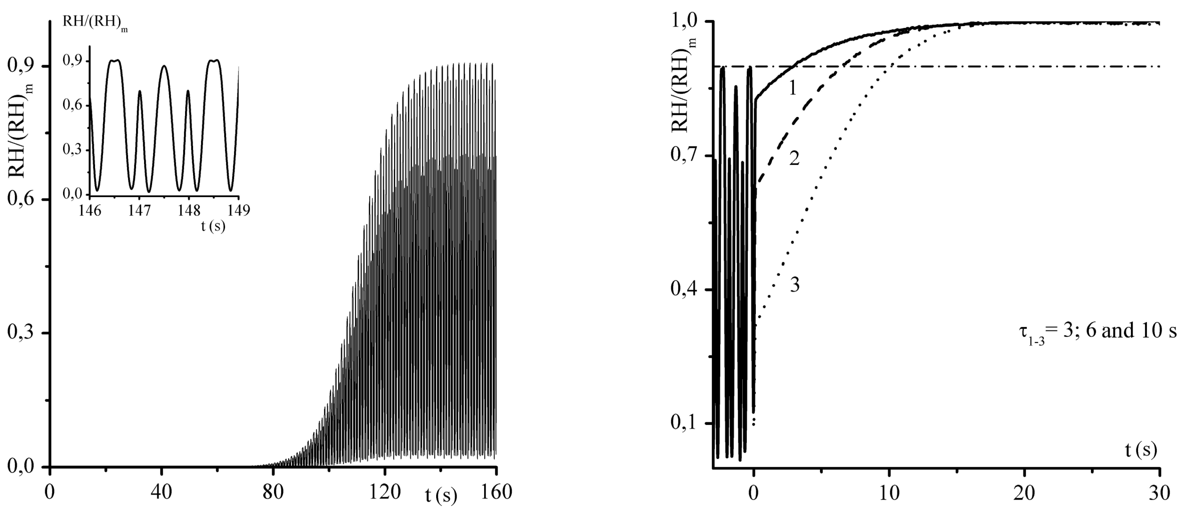

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Fig.5. SPPC mirror training by a beam with the phase of $M(x)$ oscillating in time ($\Lambda_m = 100$ μm, the amplitude and period of phase oscillations are 2π and 2 s). Transient processes for RH at the training stage (a) and after the start (b). During the start, the phase of $M(x)$ shifts by $\pi/4$ and Λ_m changes [$\Lambda_m = 100$, 90, and 80 μm for curves (1), (2), and (3)].

It is easy to see that dynamic training procedure described above is not the only one at least because the phase φ of information modulation $M(x)$ of the signal can be varied in time in many different ways. As another variant of such training, we considered the situation with rapid oscillations of φ. In this case, the input signal amplitude was modulated by the function $M(x,t)=\sin\{2\pi[\kappa_m x+\sin(f_m t)/2]\}$ with the same values f_m = 0.5 Hz and $\Lambda_m=100$ μm. Results of our simulation are presented in Figure 5. One can easy to check that in this case the mirror training stage is much shorter and takes now less than 120 s (Figure 5a). The amplitude of RH forced oscillations proves to be even larger and the oscillations are no longer harmonic (see inset in Figure 5a). Transient process for RH observed after such a training for a Gaussian beam with jumps in φ (by $\pi/2$) and Λ_m [$\Lambda_m=100$, 90, and 80 μm for curves (1), (2), and (3)] at the start moment (t = 0) is shown in Figure 5b. One can easy to check that now RH achieves 90% of its maximal value $(RH)_{max}$ for even a shorter time τ ($\tau_{1-3}=3$, 6, and 10 s for $\Lambda_m=100$, 90, and 80 μm) and the advantage in τ obtained due to the mirror training is still larger.

Substantial difference between two considered above the dynamic training procedures is illustrated in Figure 6. Here, the distributions of $\delta\eta(x,z)$ (Figures 6a,b) for the central cross-section $z=L/2$ of dynamic holograms written in PRC after the end of the mirror training by a beam with a linearly increasing (Figure 6a) and oscillating (Figure 6b) phase $\varphi(t)$ of $M(x)$ are shown by solid lines (1). Dashed lines (2) correspond to the steady-state distributions of $\delta\eta(x,z)$ written in PRC after the start of phase conjugation of the signal with $\Lambda_m=100$ μm. It is easy to check that two considered procedures give quite different results. During the mirror training by a Gaussian beam with oscillating phase, dynamic holograms written in PRC almost perfectly correspond to the refractive index gratings that should be produced at the phase conjugation stage (Figure 6b). In our opinion, this is explained by the fact that spatial and temporal harmonics of the training signal in a beam with oscillating phase are well 'mixed', which enables one to store in PRC a considerably greater amount of useful information. This is clearly demonstrated by spatiotemporal spectra $I_{f,\kappa}(f,\kappa)$ of the training field intensity presented for two considered above cases in Figures 8c,d. One can easy to see that in the case of a Gaussian beam with oscillating phase, dependence $I_{f,\kappa}(f,\kappa)$ contains much more spatial and temporal harmonics. However, in our opinion, it is much more important that due to the mixing of spatial and temporal harmonics of training field, PRC inertia (that is selection of harmonics at zero frequency) does not prevent writing the information on the expected value of Λ_m (see Figure 8d).

5. CONCLUSIONS

Our simulation of a loop self-pumped PC mirror based on a $BaTiO_3$ photorefractive crystal has shown that the time required for formation of a phase-conjugated wave in SPPC mirror can be considerably reduced by the use of its preliminary training. This is achieved by irradiating the mirror by an auxiliary (training) spatially modulated optical field, which writes static refractive index gratings in PRC before the arrival of signal radiation. These gratings provide subsequent fast (6-20 times faster) formation of the dynamic holograms required for efficient phase conjugation. Of course, to realize such procedures (to choose the optimal spatial structure of the training field), it is necessary to know some properties of the signal radiation whose wave front is to be conjugated.

Our study of several static and dynamic preliminary training procedures has show that efficiency of dynamic procedures based on temporal averaging (i.e., the use of inertia of PRC nonlinear response) are considerably higher than efficiency of static procedures because the efficient use of an information on the expected spatial period of the signal radiation.

It seems that our approach can be also applied for solving the pattern recognition problems [25,26], in the development of optical correlators [27] and elements of the associative memory [28-30] based on SPPC mirrors.

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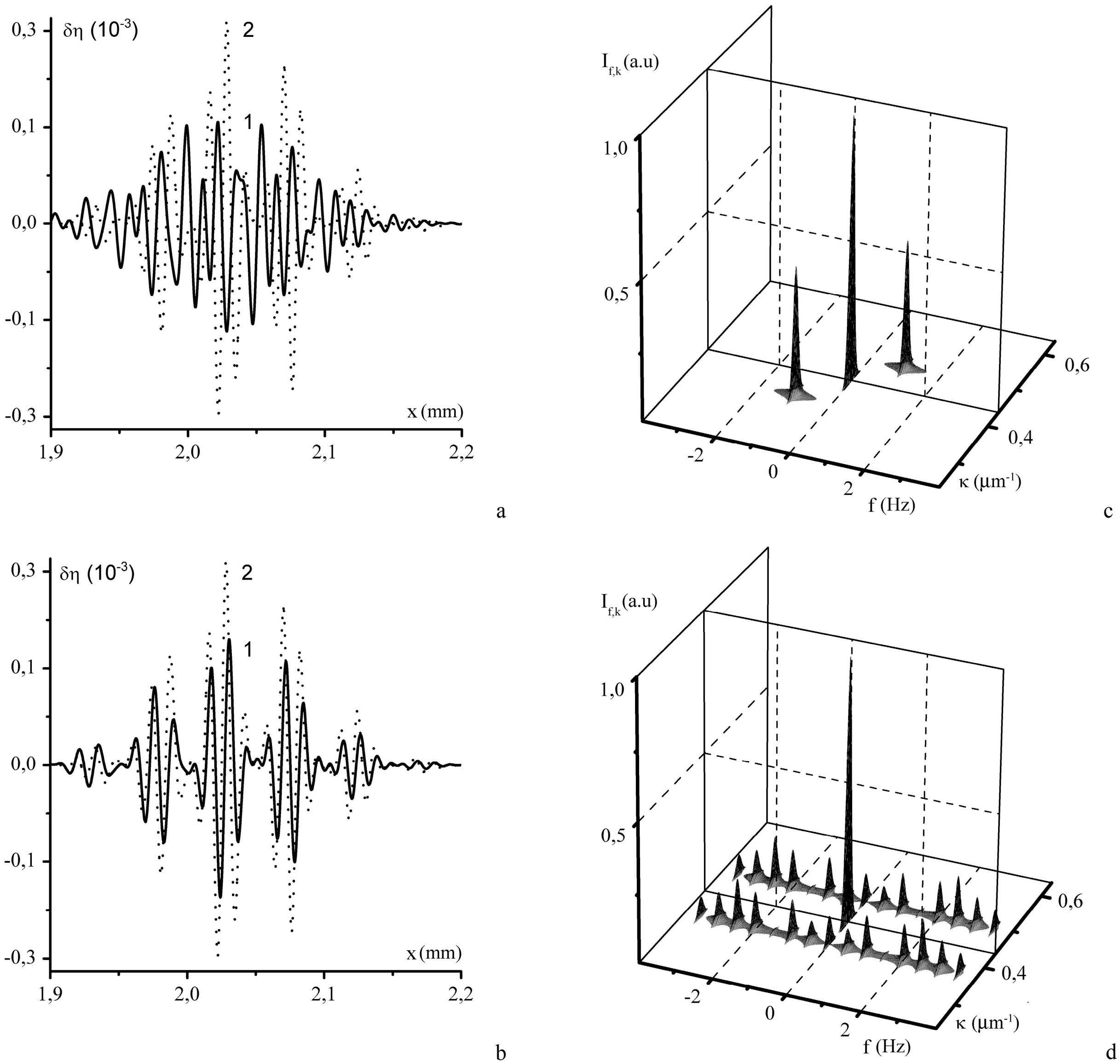

Fig.6. Training by a beam with linearly increasing (a,c) and oscillating (b,d) phase of $M(x)$ for $\Lambda_m = 100$ μm. Distributions of $\delta\eta(x, z = L/2)$ (a,b) after the end of training (1) and after the start (2). Spatiotemporal spectra of training field intensity $I_{f,\kappa}(f,\kappa)$ (c,d); the point $\kappa = 0$ corresponds to the convergence angle of waves $A_{f,b}$.

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