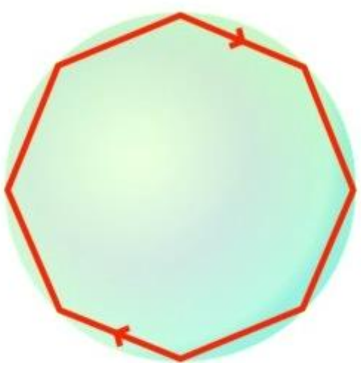
НЕЛИНЕЙНЫЕ И КВАНТОВЫЕ ЭФФЕКТЫ В ОПТИЧЕСКИХ МИКРОРЕЗОНАТОРАХ

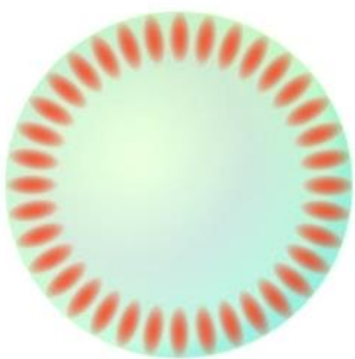
А.Н. Данилин[1,2], А.Е. Шитиков[1], Н.Ю. Дмитриев[1], Д.А. Чермошенцев[1], В.Е. Лобанов[1], Ф.Я. Халили[1], И.А. Биленко[1,2*]

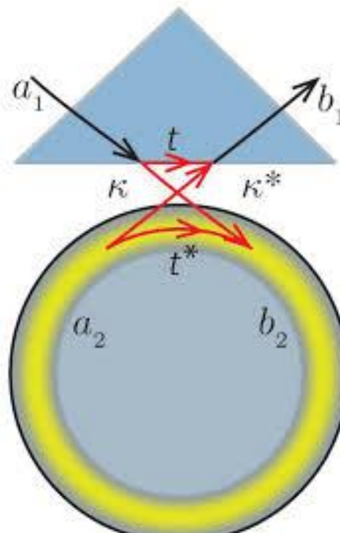
[1]*Российский Квантовый Центр*

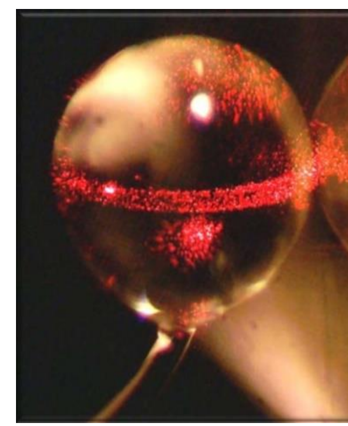
[2]*МГУ имени М.В. Ломоносова*

Whispering gallery modes





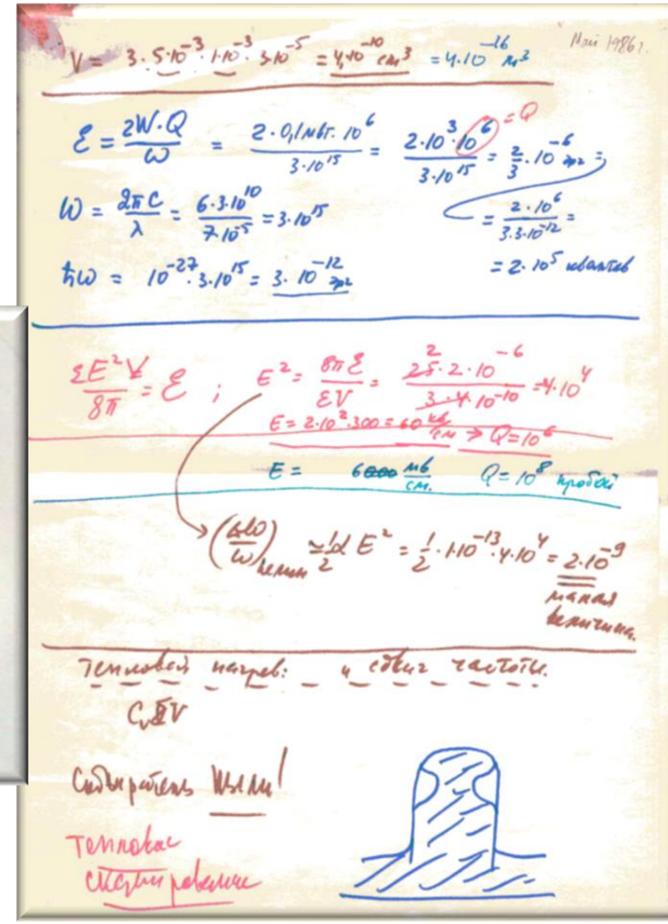




WGM resonators are multimode ones in essence. If absorption and scattering are reduced – Q factor can be very high!

Whispering gallery modes – a retrospective

Vladimir Braginsky
(1931-2016)



1986: ***"We managed to made a microwave WGM resonators with Q>10⁸. Why non to try optics?"***

Whispering gallery modes – a retrospective

Ultimate Q of optical microsphere resonators

M. L. Gorodetsky, A. A. Savchenkov, and V. S. Ilchenko

We demonstrate the quality factor $Q = (0.8 \pm 0.1) \times 10^{10}$ of whispering-gallery modes in fused-silica microspheres at 633 nm, close to the ultimate level determined by fundamental material attenuation as

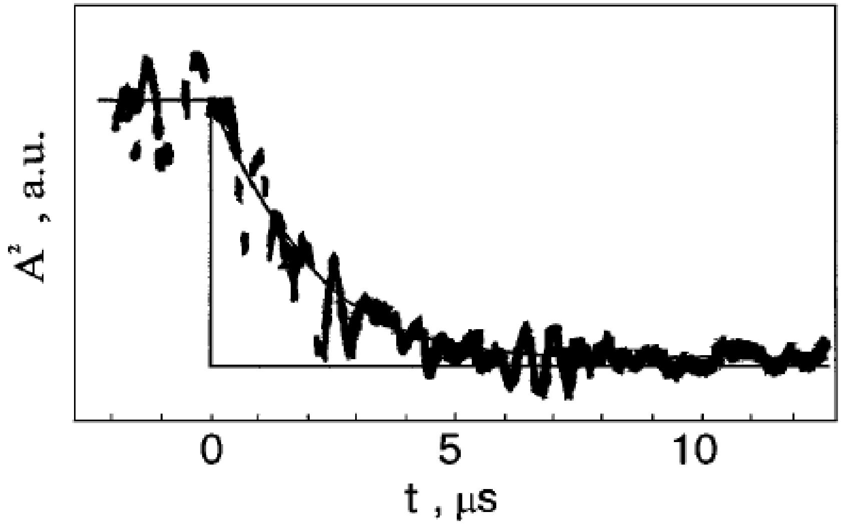


Fig. 1. Mode energy damping curve for a WG mode in a 750-μm sphere. Estimated damping time $\tau = 2.7\ \mu$s; $\lambda = 633$ nm.

Optics Letters 21(7), 453-455 (1996)

Cited: 1409

Whispering gallery modes – a retrospective

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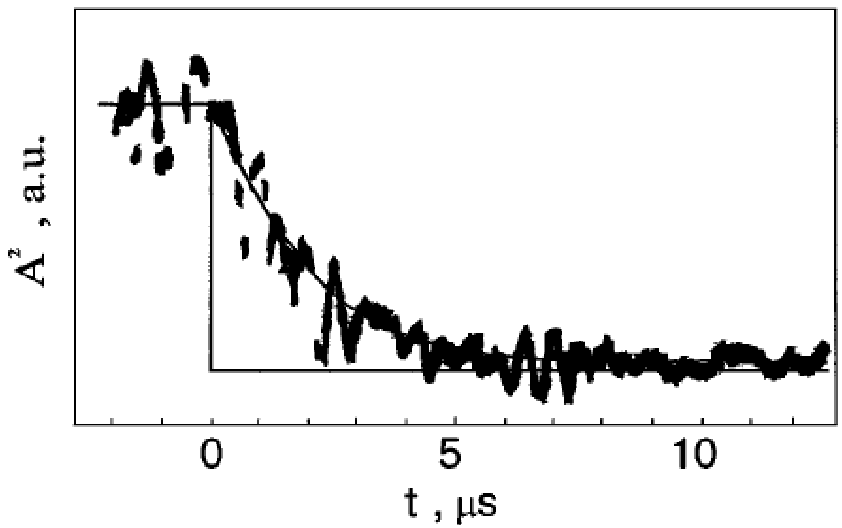

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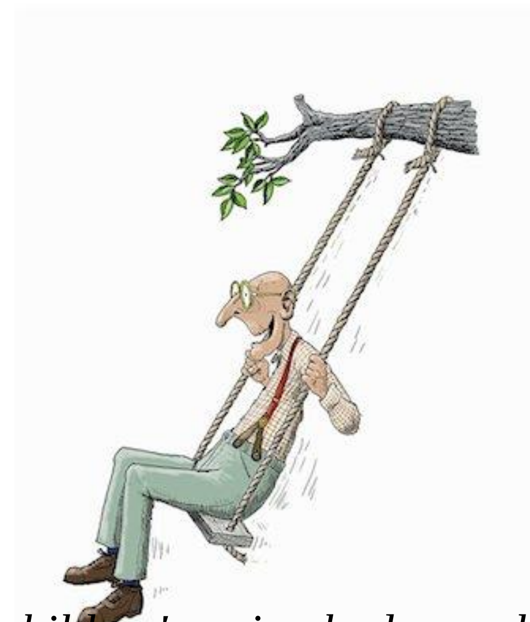
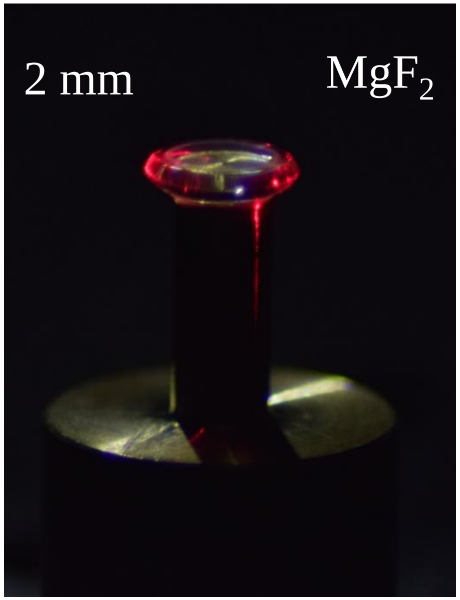
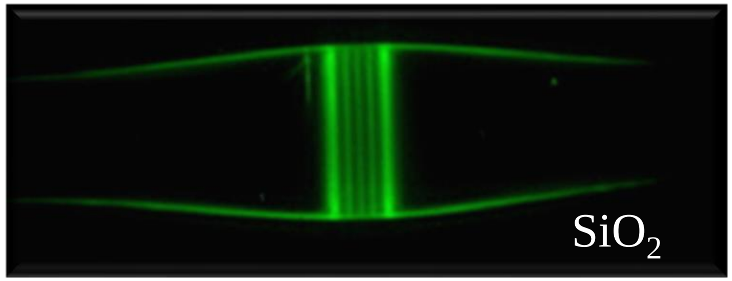
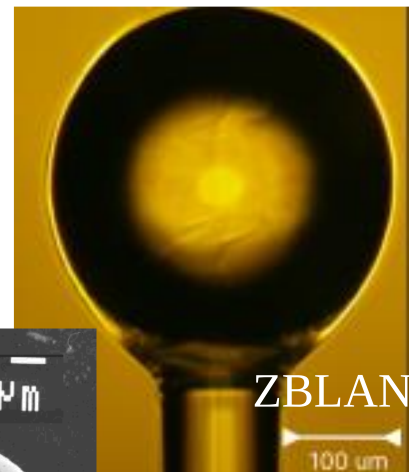


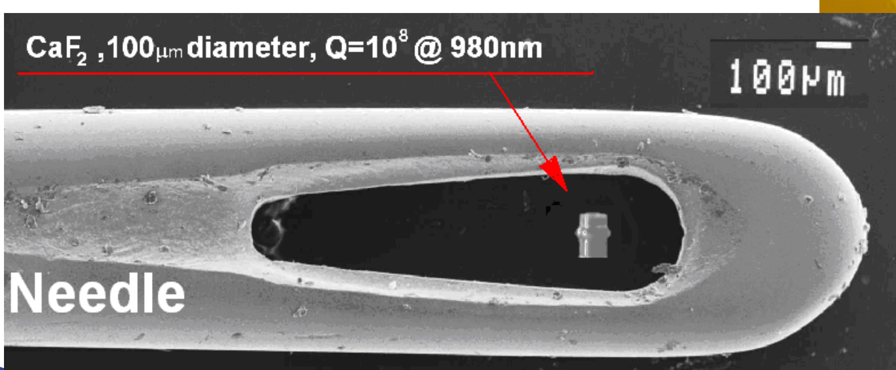
If a children's swing had a quality factor Q ~ 10^9, once pushed one could swing for 30 years!

Volumetric resonators: crystals and glasses







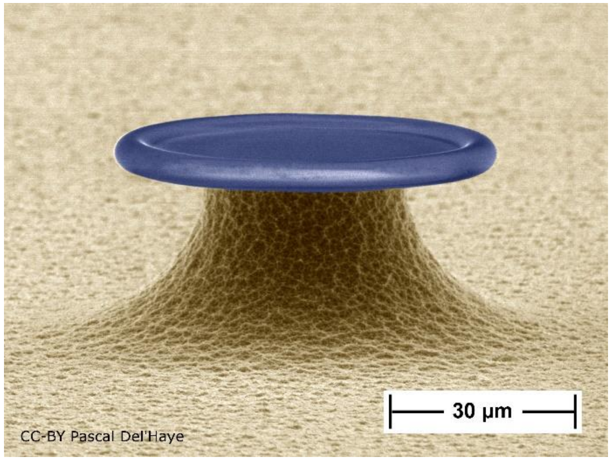


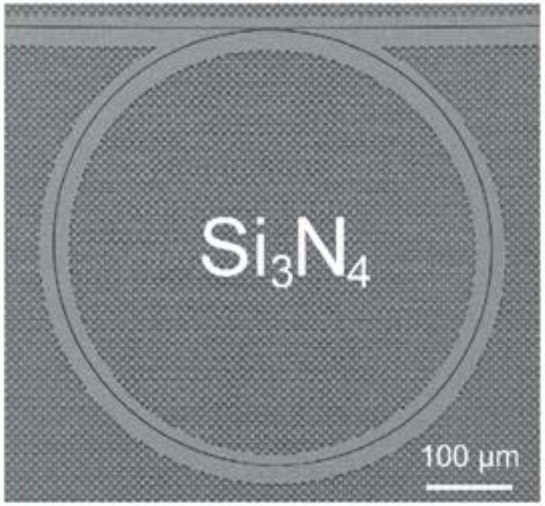
Made by diamond cutting and asymptotic polishing or by melting

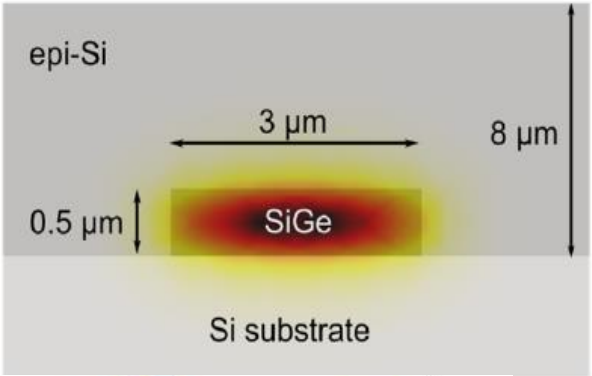
Integrated high-Q resonators

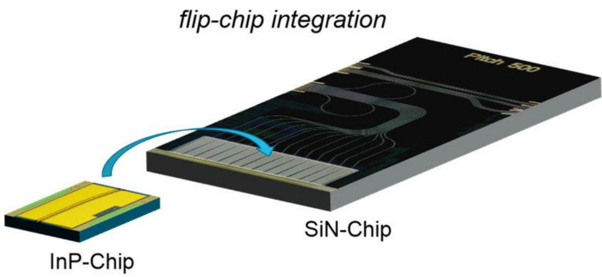
Materials:

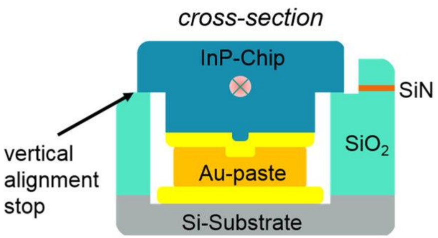
1. SiO2 (etching + reflow)
2. CMOS comparable:
   a. Si (but TPA at 1.55 μm)
   b. Si_3N_4
   c. AlN
   d. SiGe
3. InP (no high-Q but emitter complementarity)
4. LiNbO3
5. Combinations, others.

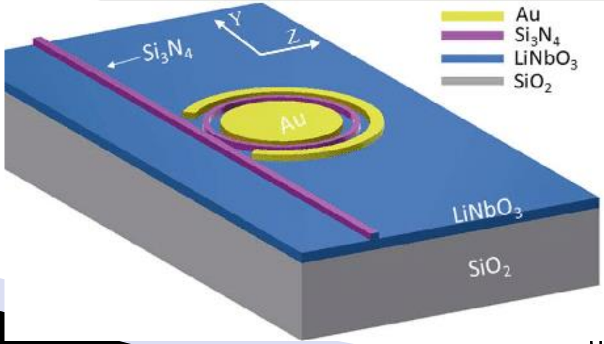












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   b. Si_3N_4 – n=2, 4 – 0.4 μm transparency, **mature technology for applications and mass productions**
   a. AlN
   b. SiGe
3. InP (no high-Q but emitter complementarity)
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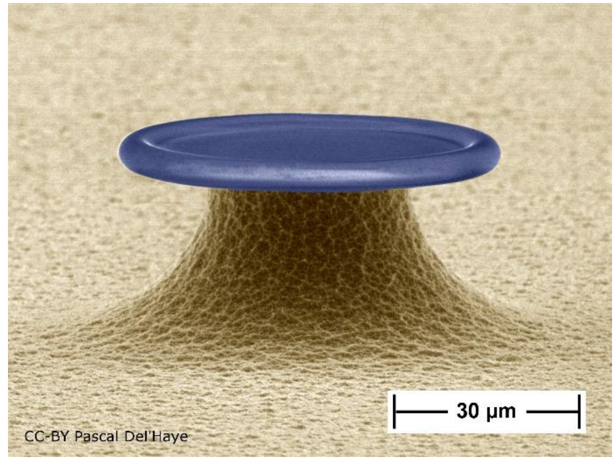

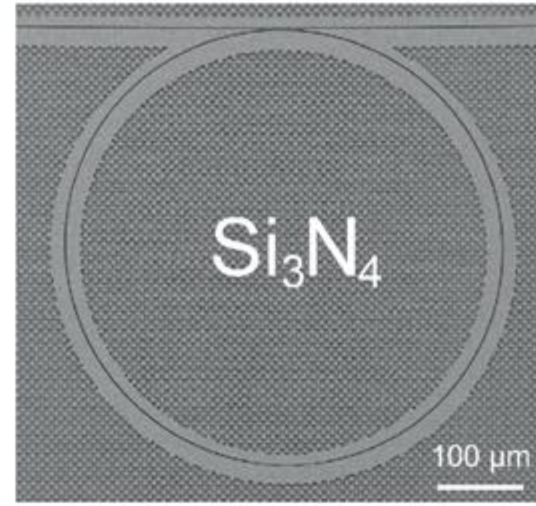

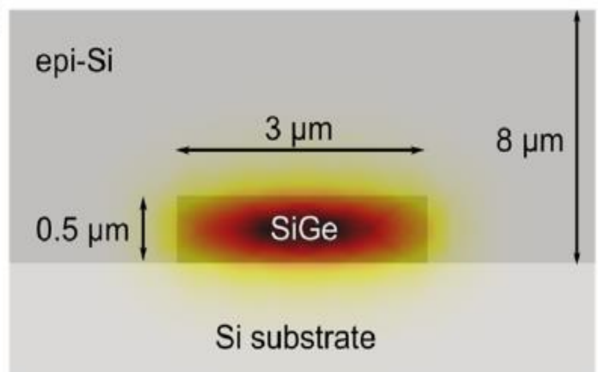

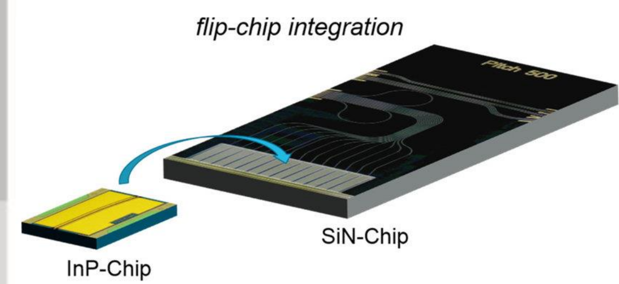

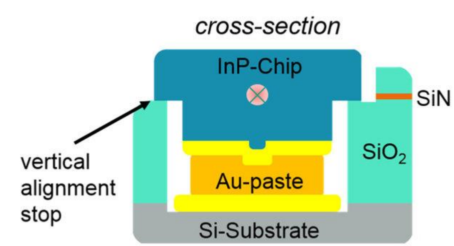

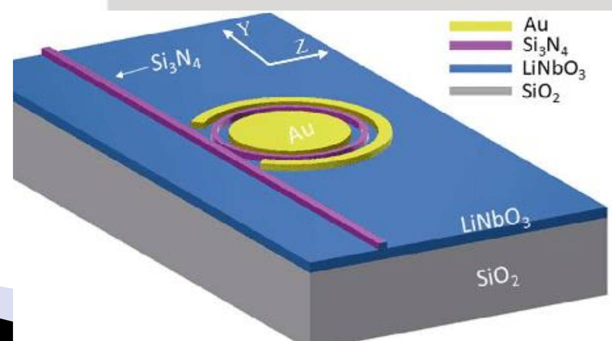

Si_3N_4 integrated high-Q resonators

	INME (Russia)	Ligentec (Swiss)	SUSTech (China)
Technology	Direct	Damascene	?
Thickness	250 – 1000 nm	300-800 nm	80-300 nm
Quality factor	3×10^6	$\cancel{5 \times 10^6}$ 3×10^7	2×10^7
Integrated heaters	Yes	Yes	~~No~~ Yes

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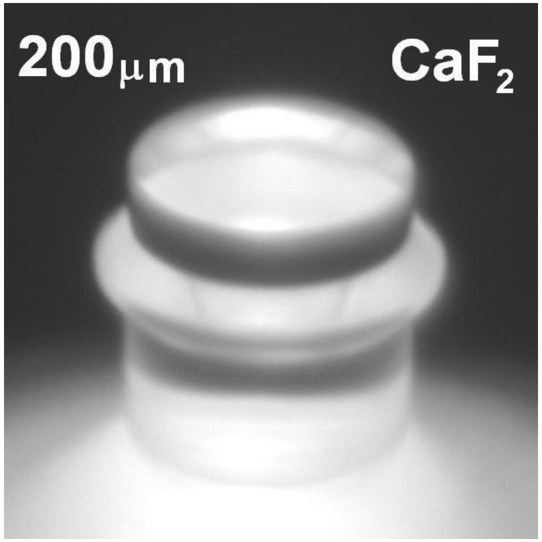
Joint research of SUSTech and RQC supported by the RSF-NSFC grant for 2023-2025

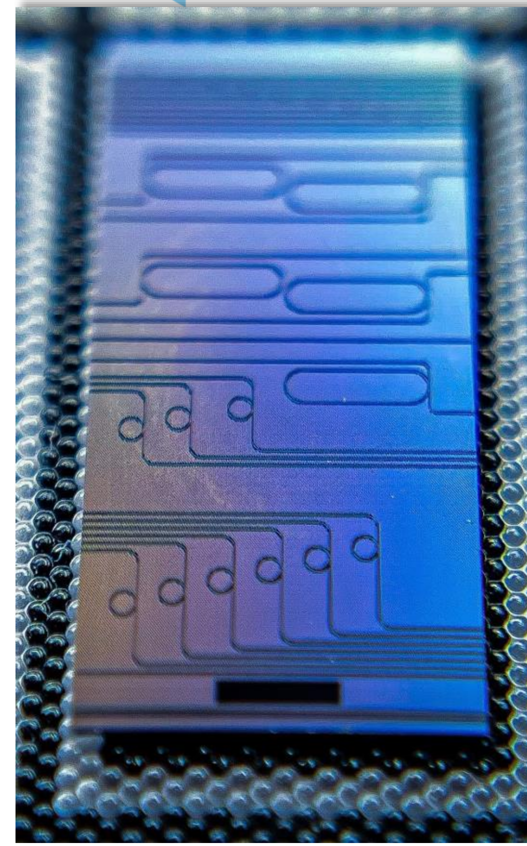
Crystalline vs integrated resonators



***Q* > 10^9**

PRO:

- Superior Q-factor
- Available from mid IR to UV wavelengths
- Tunable coupling



***Q* > 10^7** *(high confinement)*
***Q* ~ $2x10^8$** *(low confinement)*

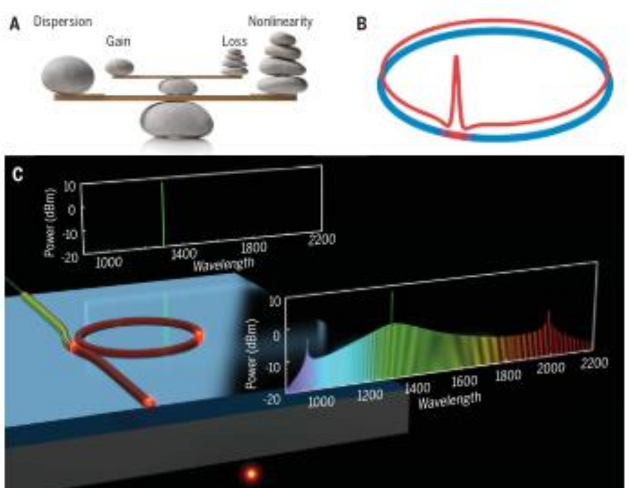
PRO:

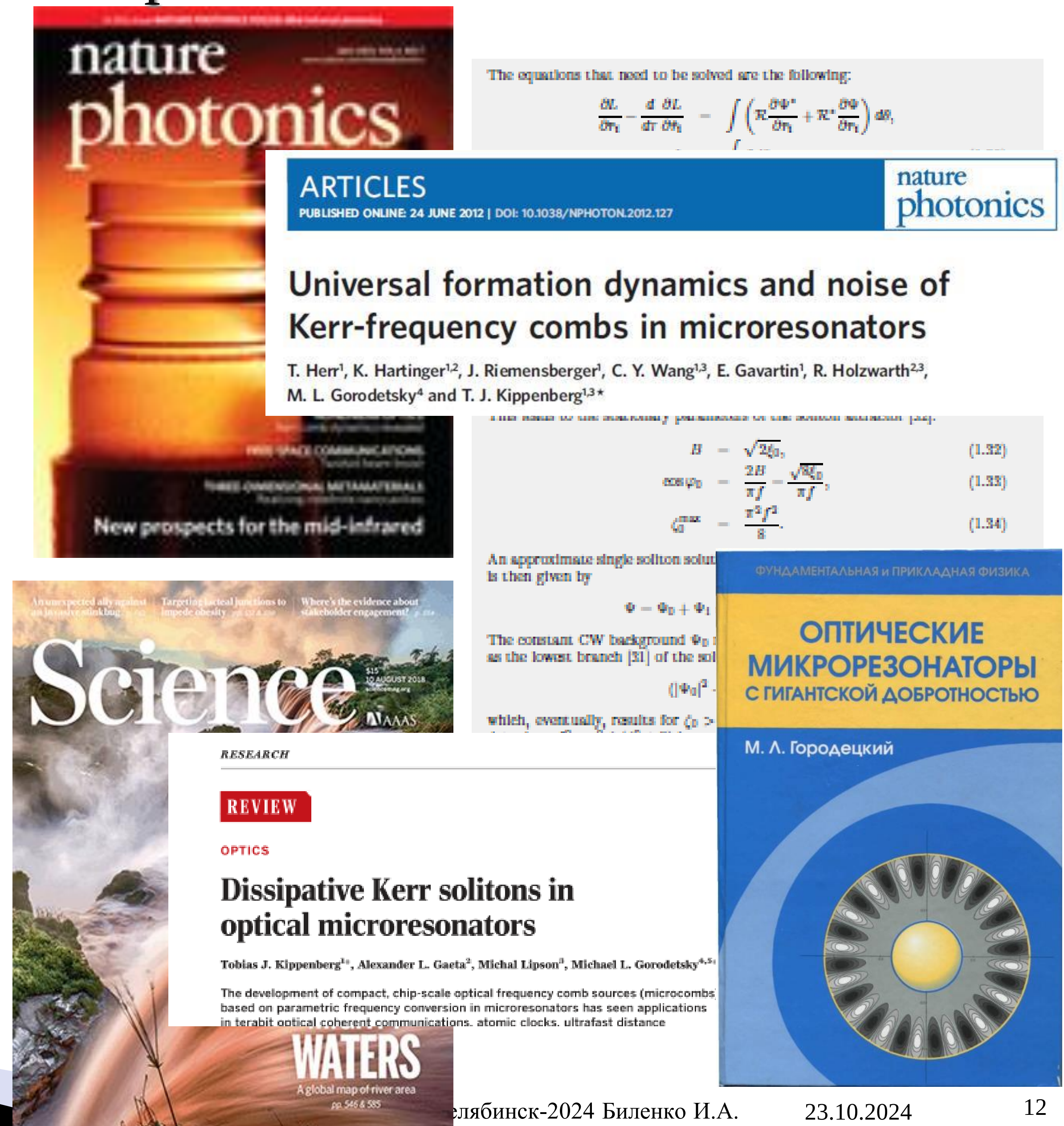
- Mass production ready
- Reproducibility
- Single mode
- Flexible and scalable design

Laboratory of Coherent Microoptics and Radiophtonics

Russian Quantum Center

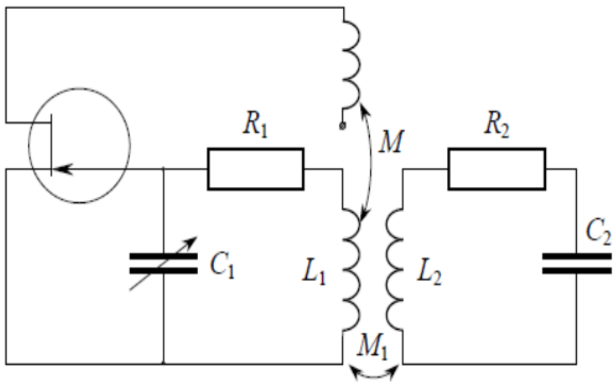
professor
Michael Gorodetsky
1966 – 2019

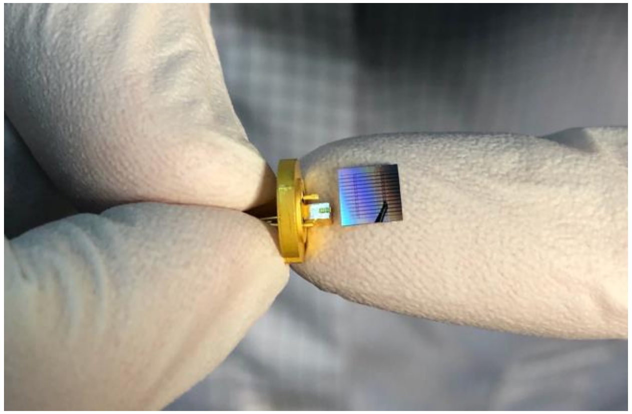


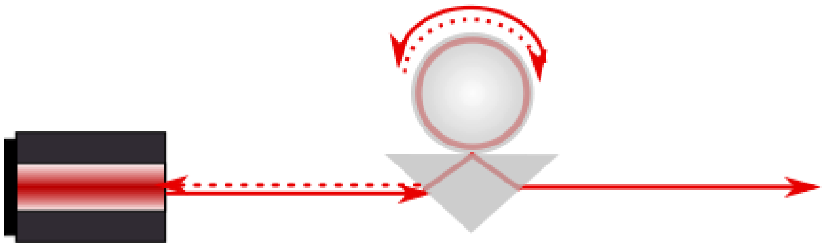


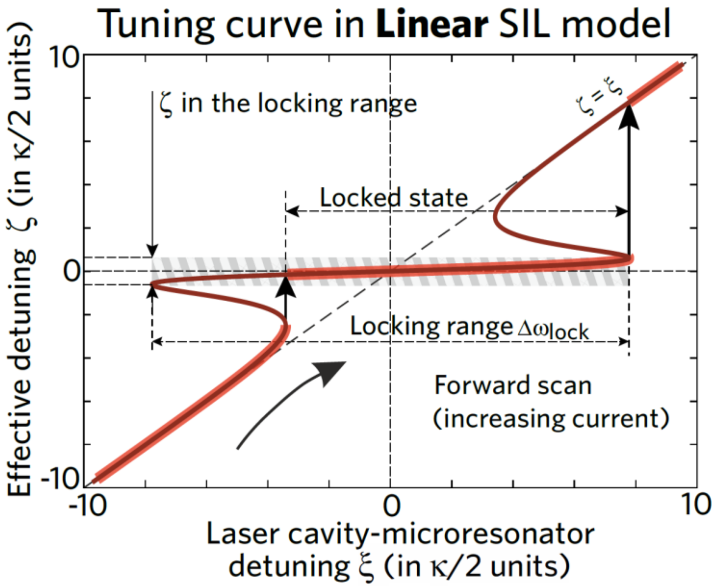
Self-injection locking (SIL) effect

Resonant Rayleigh back scattering tunes laser exactly on the WGM resonance

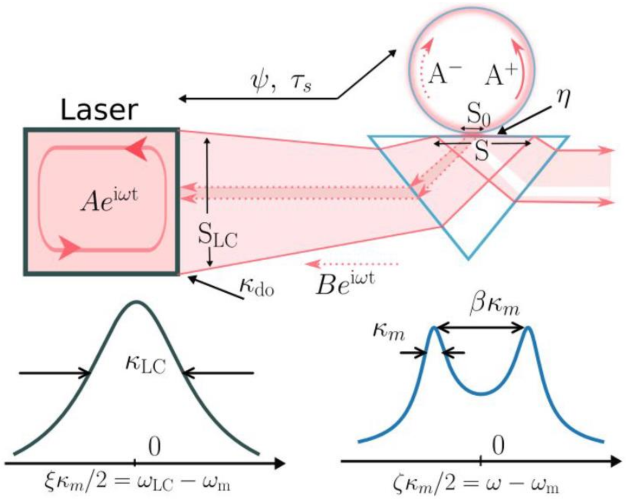


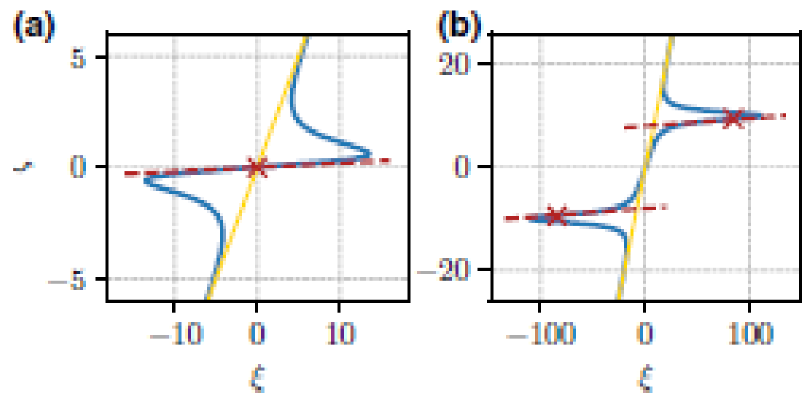


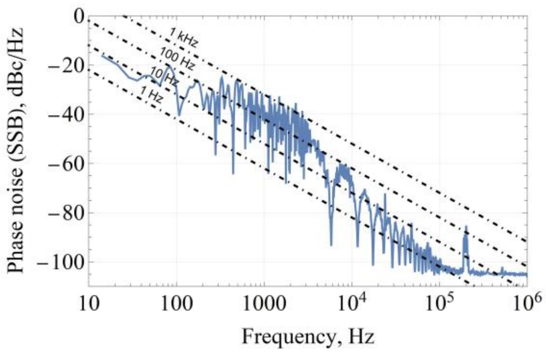


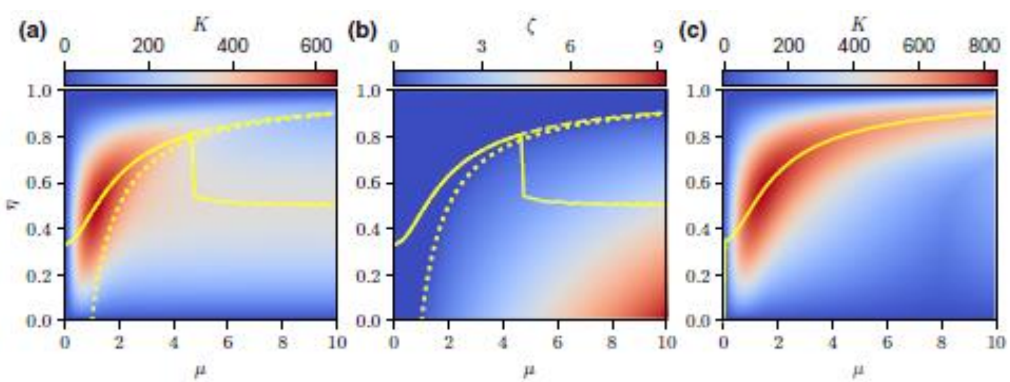


Optimization of the SIL regime







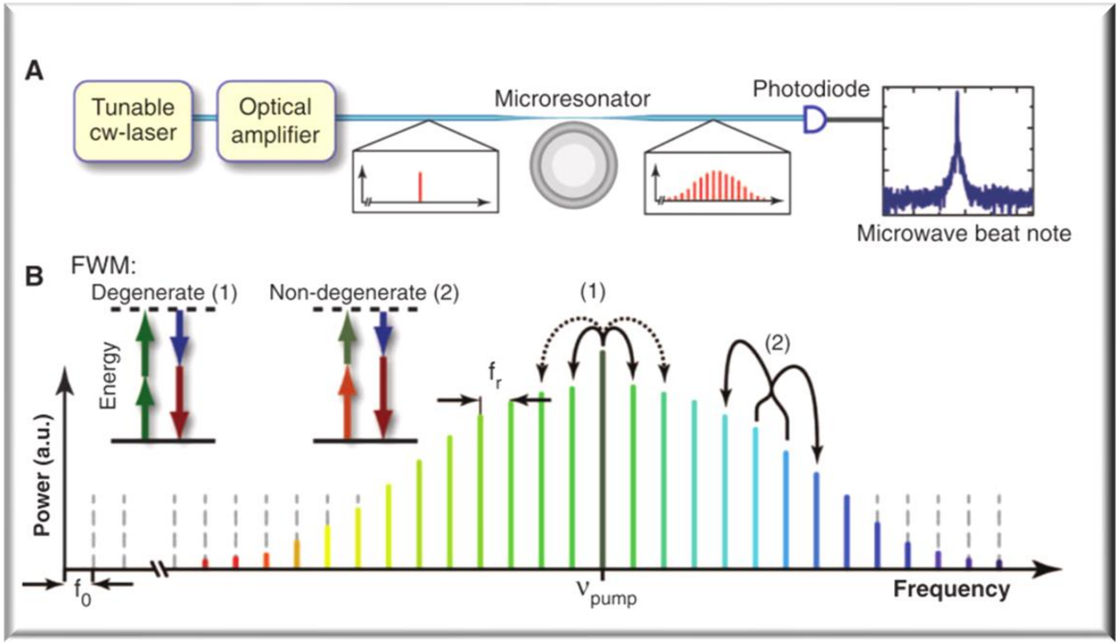


Laser line can be narrowed up to 100 times better!

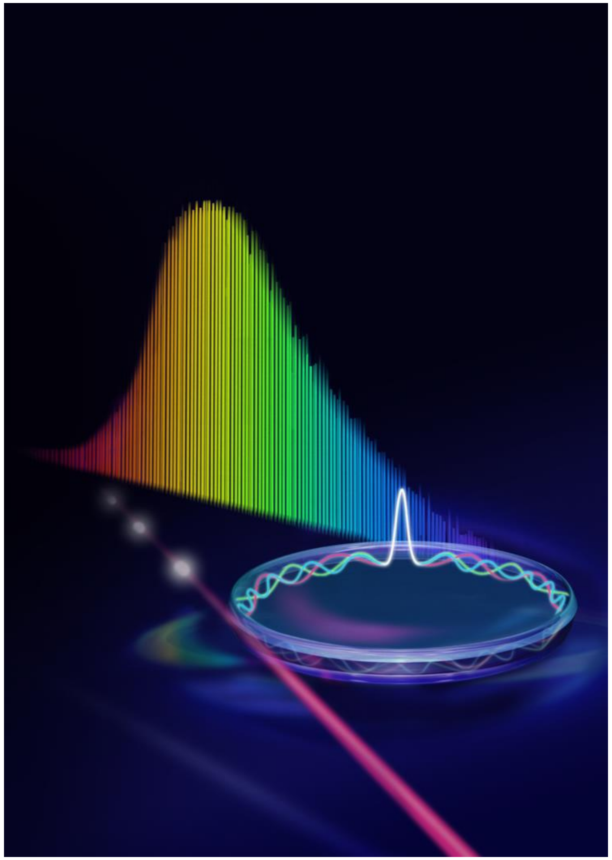
REVIEW: *Kondratiev, N. M., Lobanov, V. E., Shitikov, A. E., Galiev, R. R., Chermoshentsev, D. A., Dmitriev, N. Y., ... & Bilenko, I. A. Recent advances in laser self-injection locking to high-Q microresonators. Frontiers of Physics, 18(2), 21305, 2023.*

Generation of Kerr microcombs in microresonators

Degenerate four wave mixing threshold: $P_{thr} \propto \frac{V_m}{Q^2}$

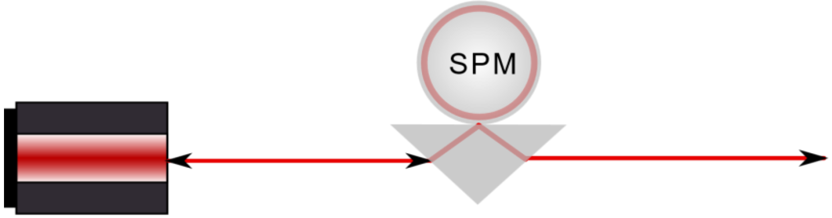


P. Del'Haye et al., 2007 (MPQ)



Effect of cubic nonlinearity: a non-linear SIL

Resonator non-linearity drastically changes the SIL dynamic:



$$\frac{dN}{d\tau} = J_N - \frac{\kappa_N}{\kappa_0} N - N g_l |A_l|^2,$$

$$\frac{dA_l}{d\tau} = \left(i\xi_0 - i v_\xi \tau + (1 + i\alpha_g) N g_l - \frac{\kappa_l}{\kappa_0} \right) A_l - e^{i\Omega_l t} \sum_\mu \tilde{\kappa}_{\text{Laser}} A_\mu^- e^{-i\omega_\mu^{(1)}(t-t_s)},$$

$$\frac{dA_\mu^+}{d\tau} = \left(-\frac{\kappa_\mu}{\kappa_0} - i d_2 \mu^2 \right) A_\mu^+ + i\beta_\mu A_\mu^- + i\tilde{g}_\mu S_\mu^+ - \tilde{\kappa}_{\text{WGR}} e^{i\omega_\mu^{(1)} t} \delta_{0\mu} A_l e^{-i\Omega_l (t-t_s)},$$

$$\frac{dA_\mu^-}{d\tau} = \left(-\frac{\kappa_\mu}{\kappa_0} - i d_2 \mu^2 \right) A_\mu^- + i\beta_\mu A_\mu^+ + i\tilde{g}_\mu S_\mu^-.$$

Kondratiev, N. M., Lobanov, V. E., Lonshakov, E. A., Dmitriev, N. Y., Voloshin, A. S., & Bilenko, I. A. Numerical study of solitonic pulse generation in the self-injection locking regime at normal and anomalous group velocity dispersion. Optics Express, 28(26), 2020.

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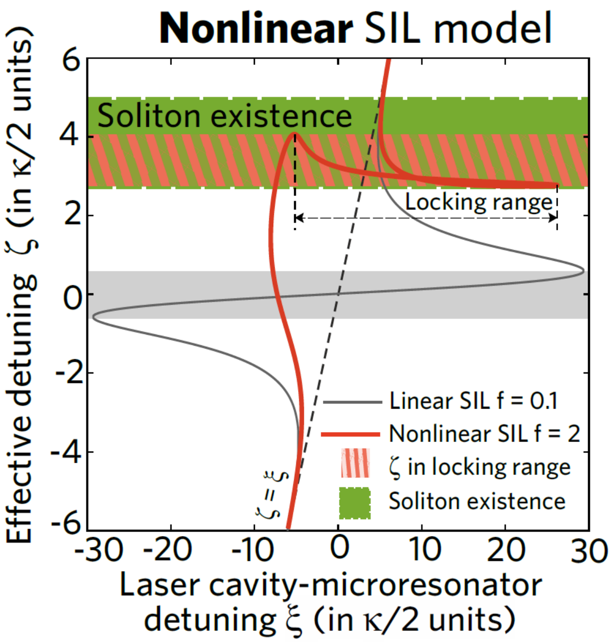
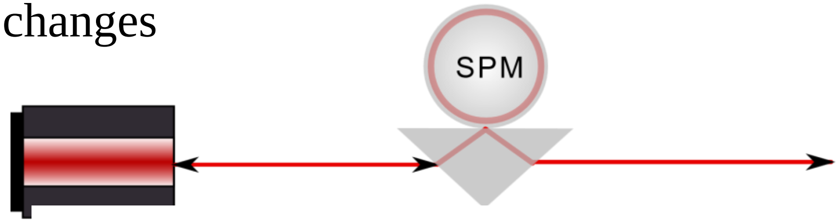
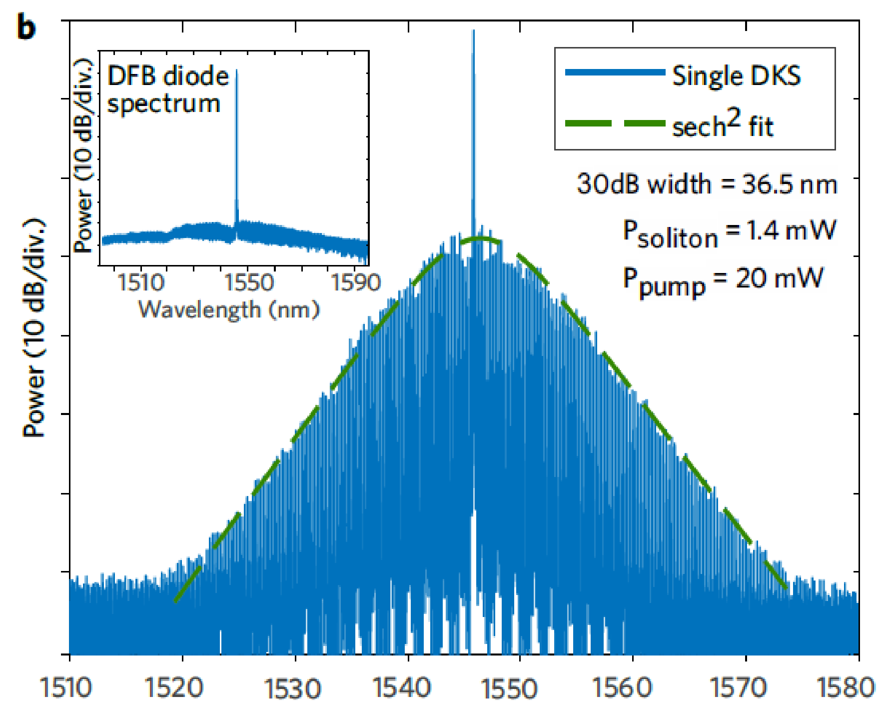
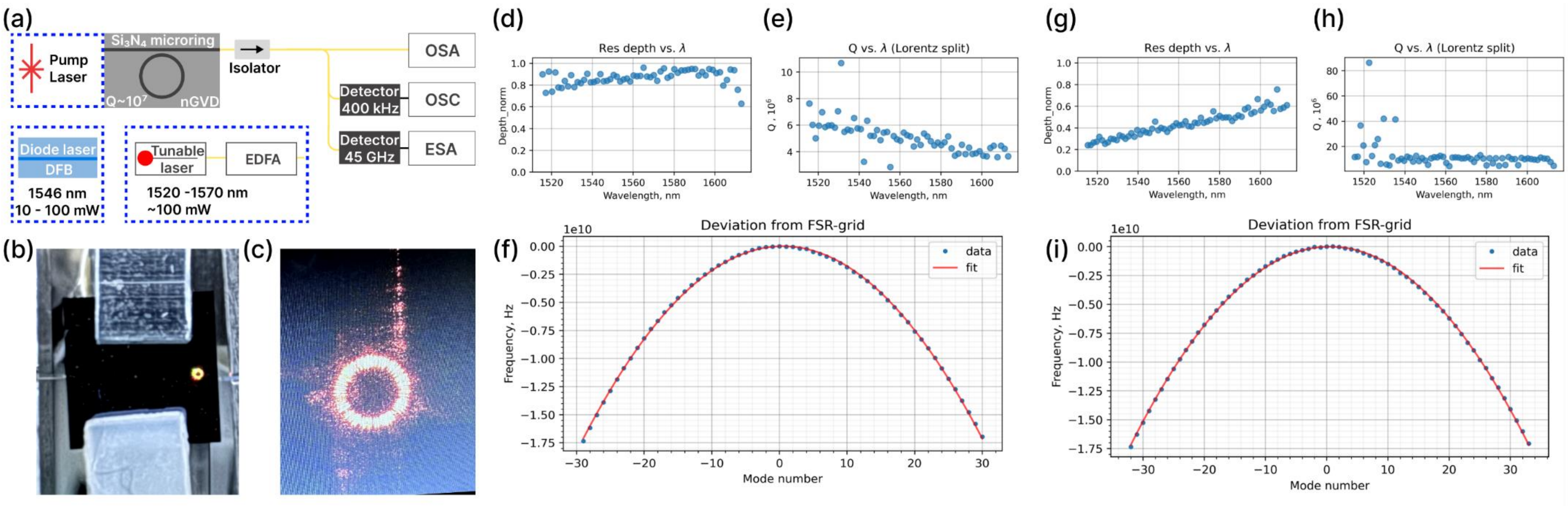






Voloshin, A. S., Kondratiev, N. M., Lihachev, G. V., Liu, J., Lobanov, V. E., Dmitriev, N. Y., ... & Bilenko, I. A. Dynamics of soliton self-injection locking in optical microresonators. Nature communications, 12(1), 1-10, 2021.

Raman solitons and platicons in Si_3N_4 integrated microring resonators



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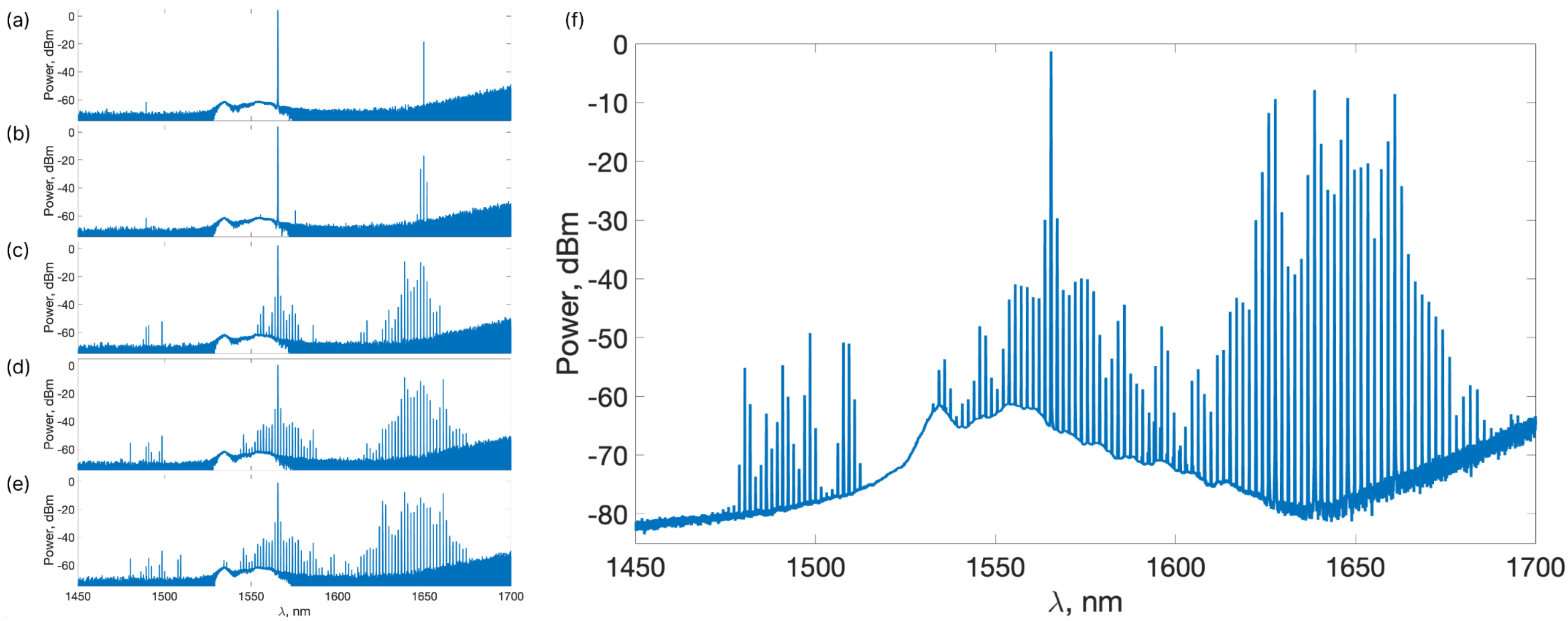
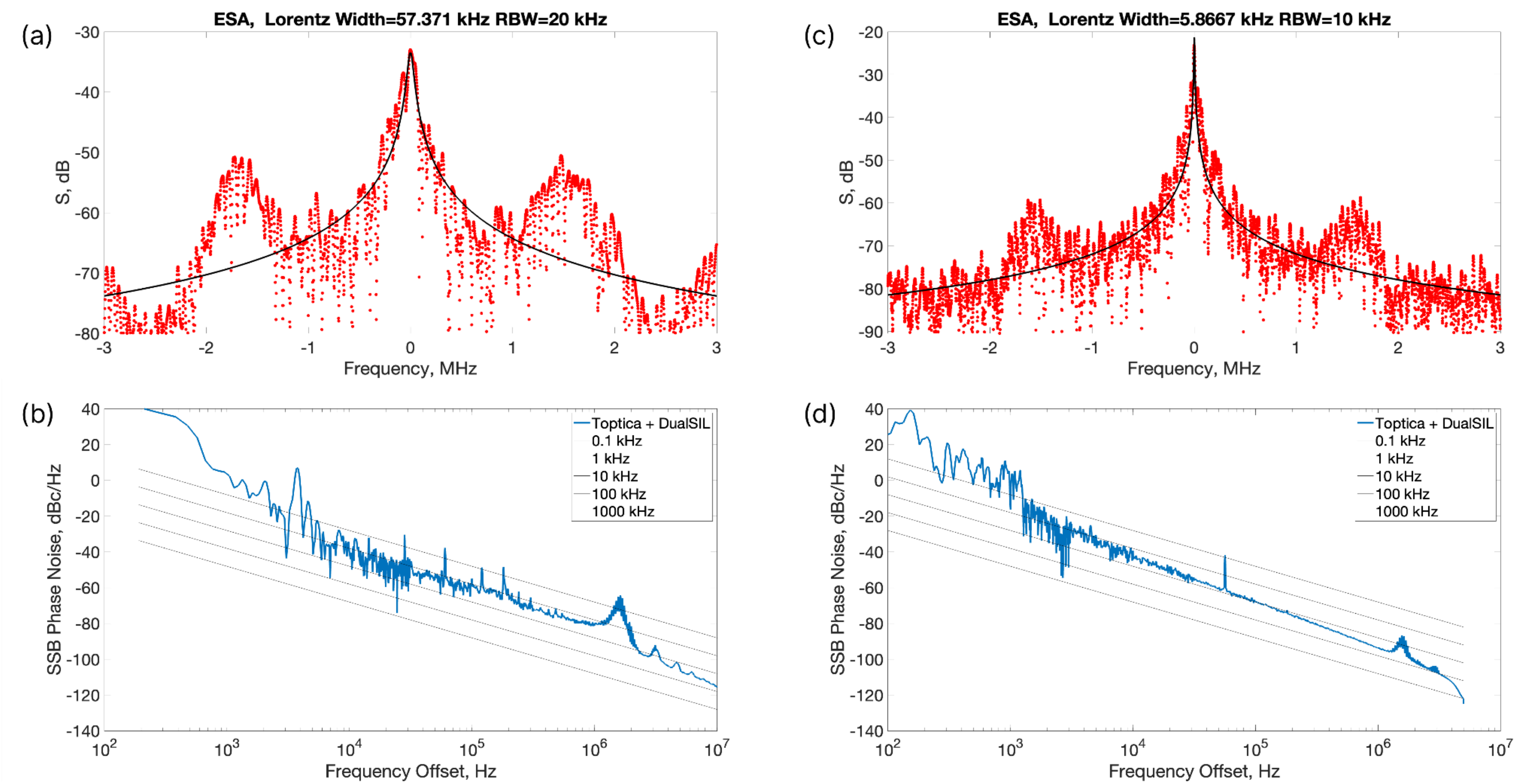
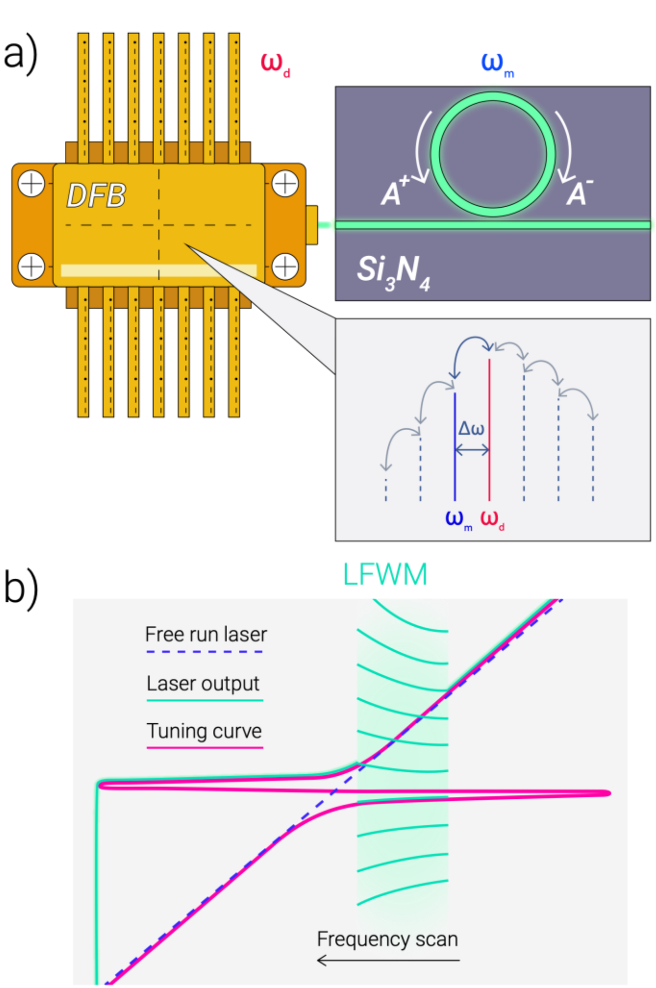


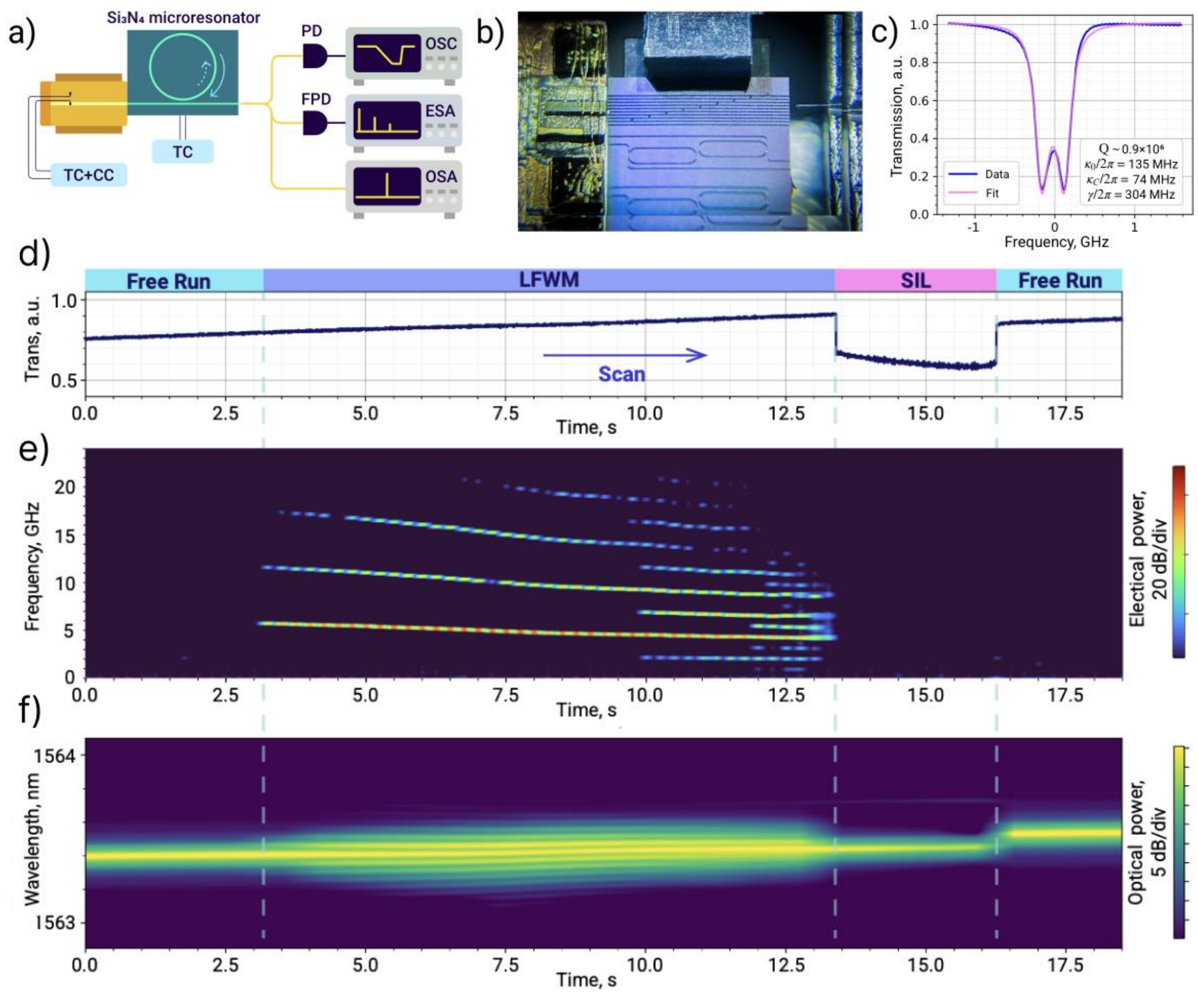
Raman solitons and platicons in Si_3N_4 integrated microring resonators

With Raman comb generation / Without Raman comb generation



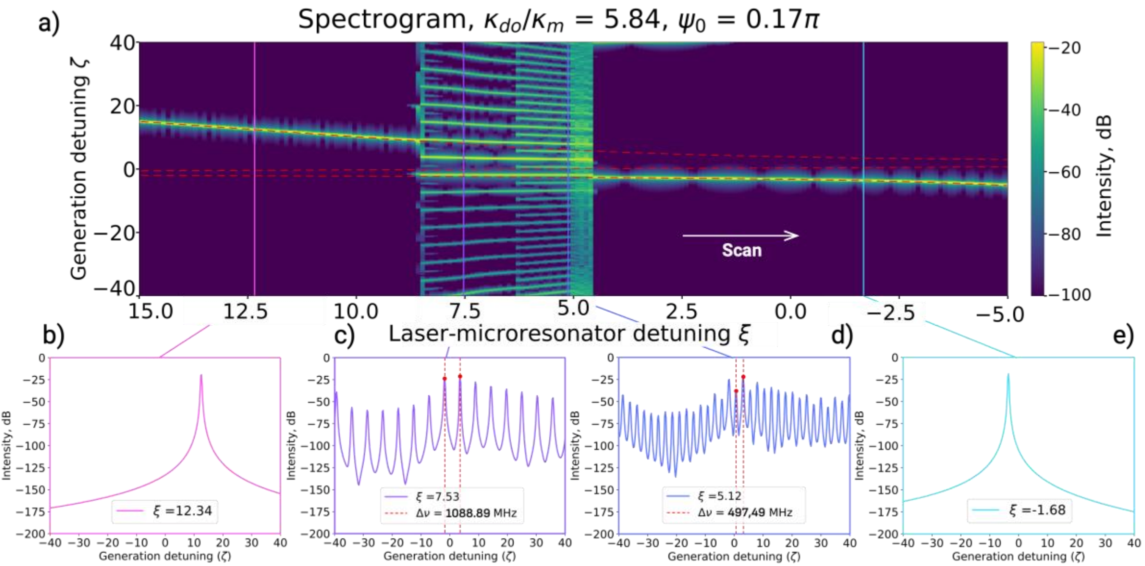
Laser Four-Wave Mixing





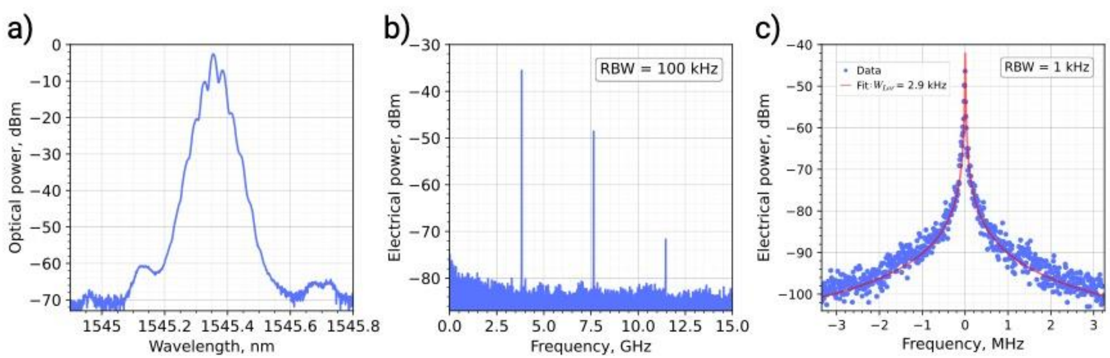
Sokol D.M. et al,. “Four-wave mixing in a laser diode gain medium induced by the feedback from a high-Q microring resonator”, Optica Open, (2024)

Laser Four-Wave Mixing



Theoretical simulation
in agreement
with experimental results

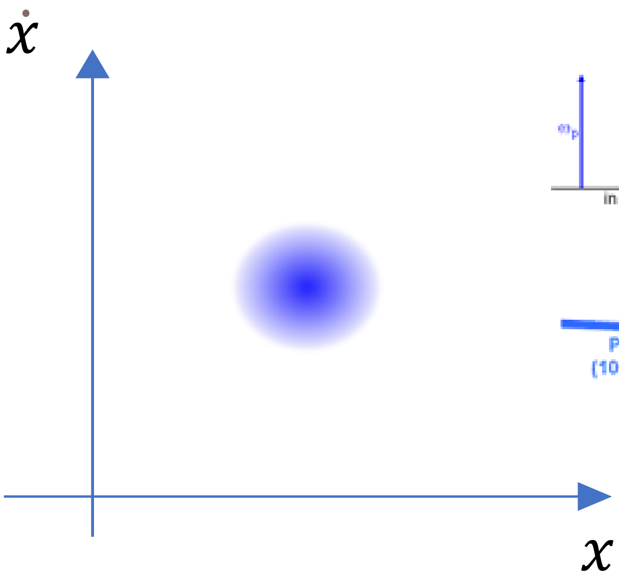
Stable RF signal generator
with low phase noise

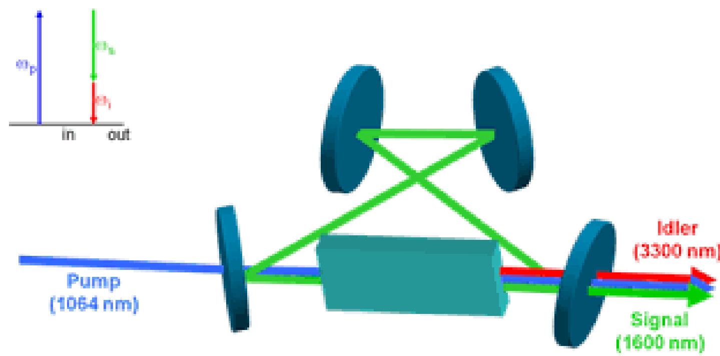


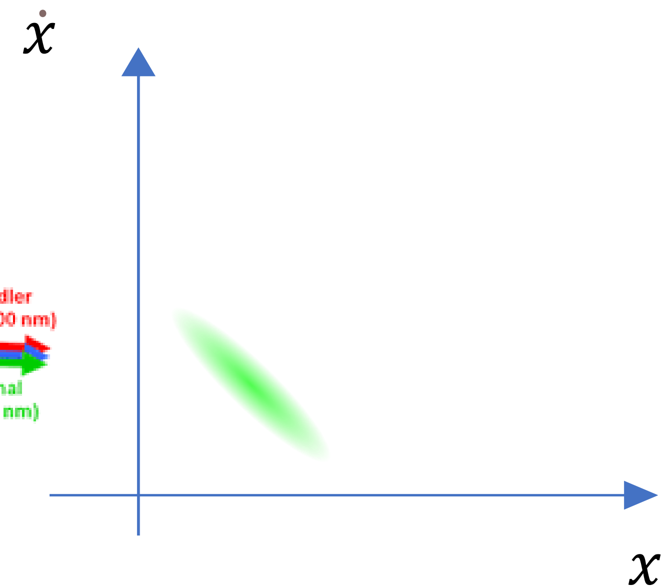
Sokol D.M. et al,. “Four-wave mixing in a laser diode gain medium induced by the feedback from a high-Q microring resonator”, Optica Open, (2024)

Effects of cubic nonlinearity – quantum squeezing

Parametric oscillation can be used for quadrature squeezing:
Weak signal - distributions still gaussian!

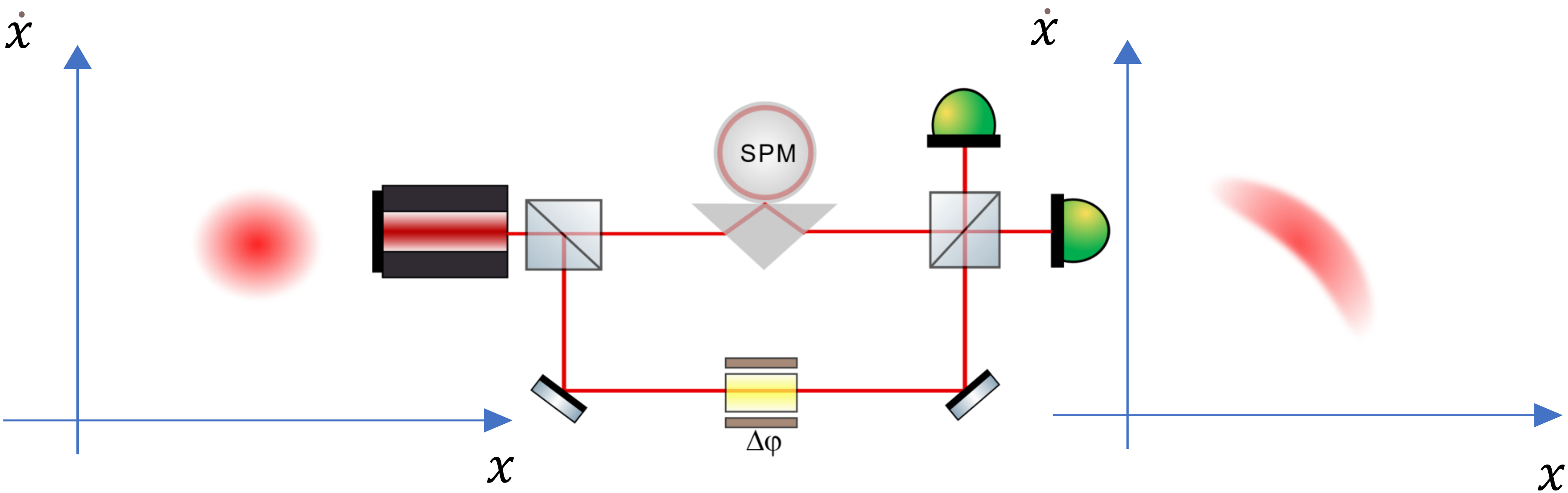






Effects of cubic nonlinearity – self phase modulation: Possibility to obtain “true” non-classical states

SPF in $\chi^{(3)}$ media could produce a **bright** “banana” state with **non-**gaussian distributions



Lumped single mode system – Hamiltonian model

$$\hat{H}_{raw} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\gamma{\hat{a}^{\dagger}}^{2}\hat{a}^{2}$$

Assuming the initial state is a coherent one $|\alpha\rangle$ with the amplitude

$$\alpha = \sqrt{\bar{n}} \gg 1$$

rotating wave approximation (mean shift due to the self-phase modulation is $2\gamma\alpha^2$):

$$\hat{a}(t) := \hat{a}(0)e^{-i(\omega+2\gamma\alpha^2)t}$$

Effective Hamiltonian for the bright state:

$$\hat{H} = \hbar\gamma\left({\hat{a}^{\dagger}}^{2}\hat{a}^{2} - 2\alpha^2\hat{a}^{\dagger}\hat{a}\right)$$

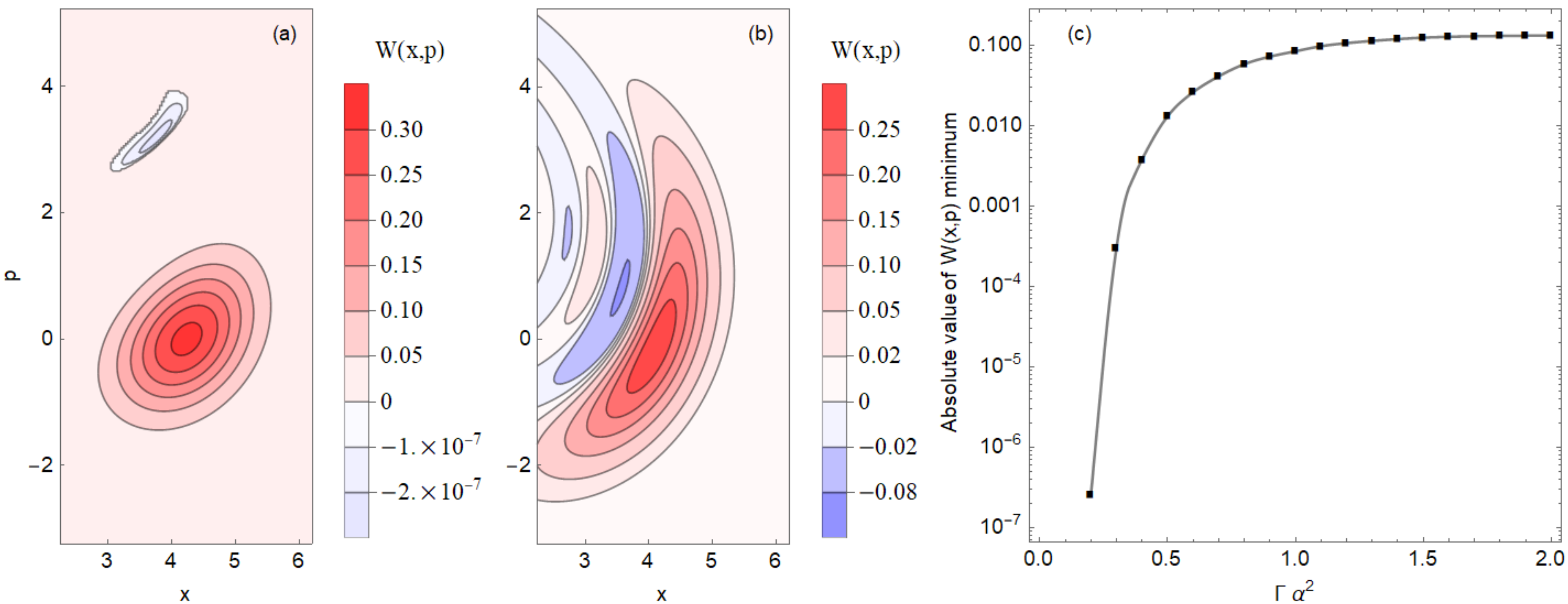
Linearized, lossless case, $\text{ù} = 2\Gamma\alpha^2$, $\Gamma = \gamma\tau$

$$(\Delta n)^2 = \frac{\alpha^2}{4\text{ù}^2} + 2\text{ù}^4 \geq \frac{3}{2^{5/3}}\,\alpha^{4/3}, \qquad \sqrt{(\Delta n_{min})^2} \sim \bar{n}^{1/3} < \sqrt{n}$$

The complete analysis gives

$$(\Delta n)^2 = \eta\alpha^2 \frac{4\text{ù}^2(1-\eta) + 1/\eta}{4\text{ù}^2 + 1/\eta} \qquad \sqrt{(\Delta n_{min})^2} < \bar{n}^{1/3}$$

Here $\eta \leq 1$is a quantum efficiency.

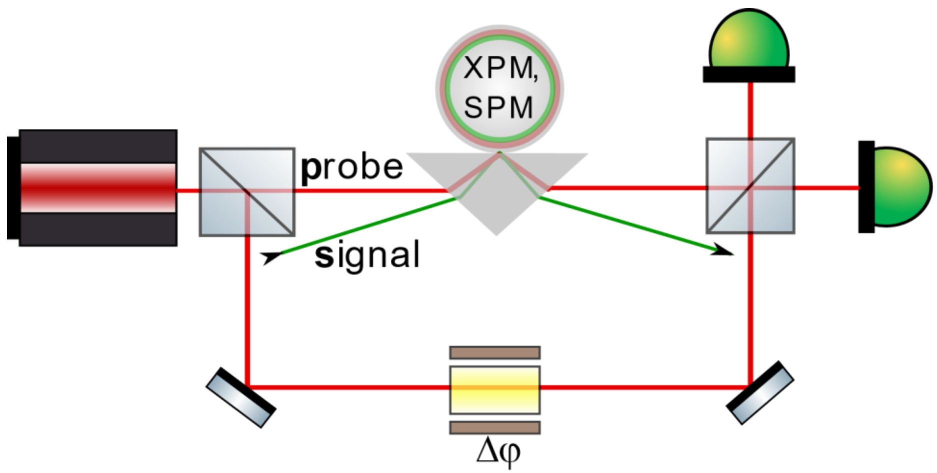


$\Gamma\alpha^2 = 0.3$ $\qquad$ $\Gamma\alpha^2 = 1$

Wigner function

Balybin, S. N., et al. Physical Review A 106.1 (2022): 013720.

Effects of cubic nonlinearity – cross phase modulation: way to photon number QND measurements



$$\phi_p(t) = \phi_p + \Gamma_S n_p + \Gamma_X n_s \,,$$

$$\phi_s(t) = \phi_s + \Gamma_S n_s + \Gamma_X n_p$$

$$\Gamma_{S,X} = \gamma_{s,x}\tau$$

$$\Delta\phi_{s\,pert} = \Gamma_X \Delta n_p$$

QND measurements – simplified analysis

Assume coherent initial quantum state of the probe mode:

$$\Delta\phi_p = \frac{1}{2\sqrt{\bar{n}_p}}\,, \qquad \Delta n_p = \sqrt{\bar{n}_p}$$

Measure not the probe phase sift but a linear combination of its phase and photon number to exclude SPF issue:

$$\phi_p(t) - \Gamma_S n_p(t) = \phi_p + \Gamma_X n_s\,,$$

In this case:

$$\Delta n_{s\,meas} = \frac{1}{2\Gamma_X\sqrt{\bar{n}_p}} \qquad \Delta\phi_{s\,pert} = \Gamma_X\sqrt{\bar{n}_p}$$

Necessary condition for QND:

$$2\Gamma_X\sqrt{\bar{n}_p}\sqrt{\bar{n}_s} > 1$$

QND measurements – Hamiltonian model

$$\hat{\mathcal{H}} = -\frac{\hbar\gamma_S}{2}\sum_{x=s,p}\hat{n}_x(\hat{n}_x - 1) - \hbar\gamma_X\hat{n}_p\hat{n}_s\,,$$

Perform a the homodyne measurement of the quadrature $\hat{X}_\zeta$ of the probe:

$$\hat{X}_\zeta = \frac{1}{\sqrt{2}}\left[\hat{a}_p(t)e^{i\zeta} + \text{h.c.}\right] = \frac{1}{\sqrt{2}}\left[e^{i(\Gamma_S\hat{n}_p + \Gamma_X\hat{n}_s + \zeta)}\hat{a}_p + \text{h.c.}\right]$$

In the case of weak non-linearity and strong probe field we can assume that

$$|\Gamma_S| \to 0\,, \qquad \bar{n}_p \to \infty\,, \qquad \text{but } \Gamma_S\bar{n}_p \text{remains finite}$$

$$\left(\Delta\hat{X}_\zeta\right)^2 = \frac{1}{2} - \Gamma_S\bar{n}_p\sin 2\varphi + 2\Gamma_S^2\bar{n}_p^2\sin^2\varphi$$

$$\varphi = \Gamma_S\bar{n}_p + \Gamma_X n_s + \zeta$$

In the presence of loss:

$$\bar{n}_p^{opt} = \frac{1}{2\Gamma_S\sqrt{\eta(1-\eta)}} \qquad \left(\Delta n_{s,min}^{opt}\right)^2 = \frac{\Gamma_S}{\Gamma_X^2}\sqrt{\frac{1-\eta}{\eta}}$$

QND measurements – steps to experiment

For a WGM microresonators:

$$\Gamma_X = 2\Gamma_S = 2Q_{load}\frac{n_2}{n_0}\frac{\hbar\omega_0 c}{V_{eff}}$$

Material	n_2, 10^{-16} cm ^{2}/W	Q_{unload}	BW, 10^6 rad/s	γ, rad/s	K $^{-1}$, dB
Al_2O_3	2.8	2×10^9	0.63	0.06	12.8
CaF_2	3.2	3×10^{11}	0.004	0.4	8
MgF_2	0.9 (e,o)	6×10^9	0.3	0.03 (e,o)	12.5
Quartz	3.4	5×10^9	0.25	0.1	11.6
Fused silica	2.6	9×10^9	0.14	0.08	11.3
$LiNbO_3$	20 (o)	10^9	1.25	0.26 (o)	12(o)
Si_3N_4	25	2×10^8	15.7	0.39	14
Si	100	10^9	1.25	0.5	11.6

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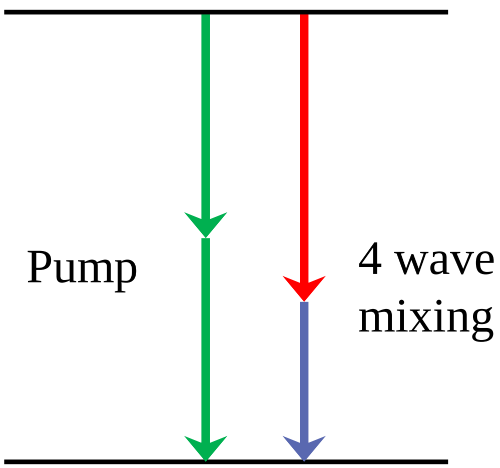
QND is possible: $\bar{n}_p^{opt} \approx 4\times10^6$ $\quad \left(\Delta n_{s,min}^{opt}\right)^2 \approx 2\times10^5 < \left(\Delta n_s^{SQL}\right)^2 = 10^6$

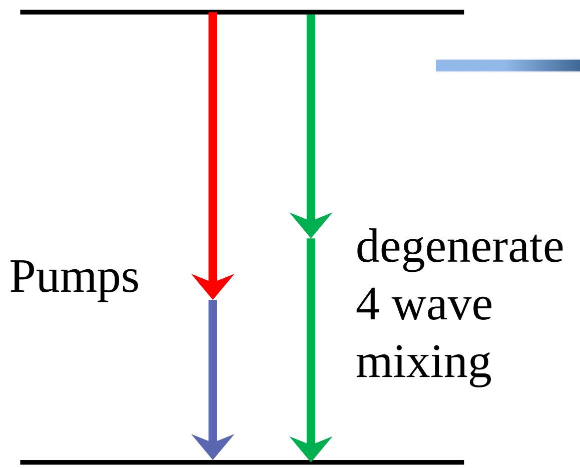
$$P_p = \frac{\hbar\omega_0^2\bar{n}_p}{2Q_{load}} \approx 0.3\ \mu W$$

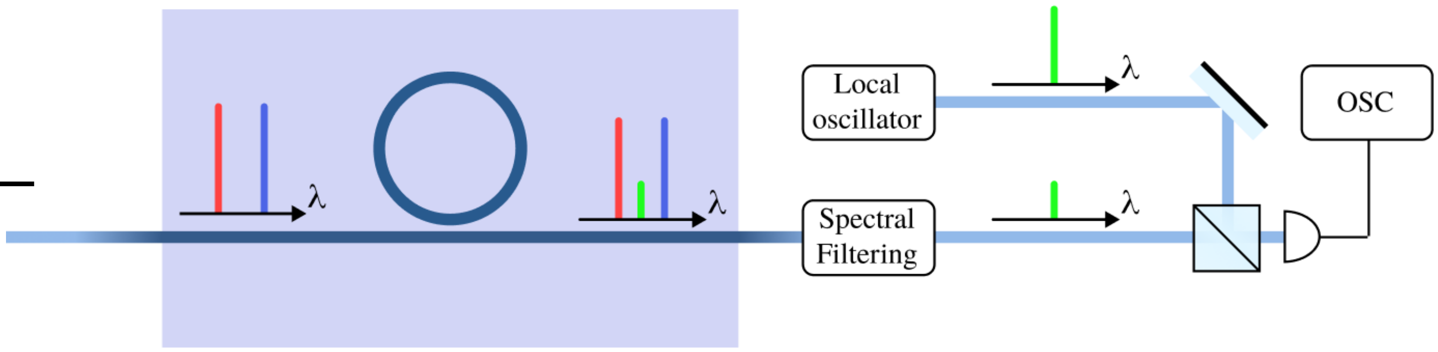
Parametric oscillations in dual-pumped microresonator and quantum light squeezing

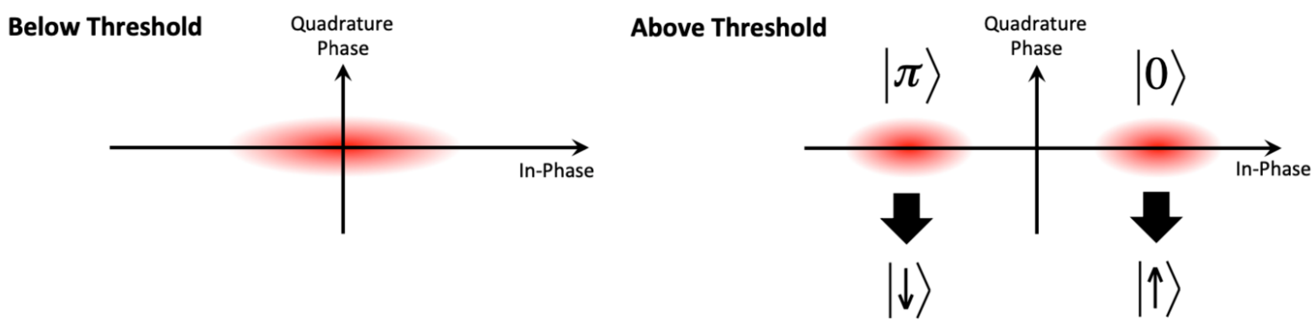


Parametric oscillations in dual-pumped microresonator and quantum light squeezing

Effects of cubic nonlinearity: Dual-pumped χ(3) degenerate optical parametric oscillator (DOPO):







Parametric oscillations in dual-pumped microresonator and quantum light squeezing

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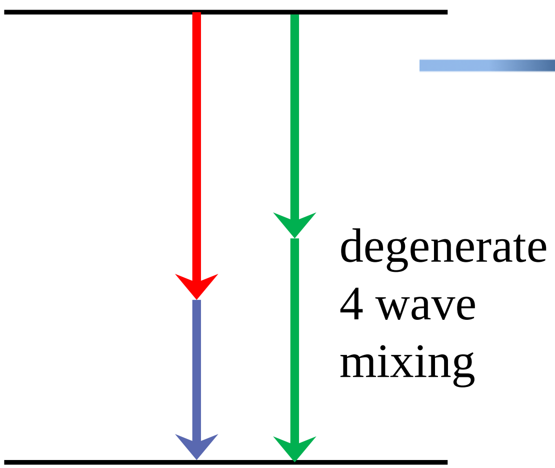

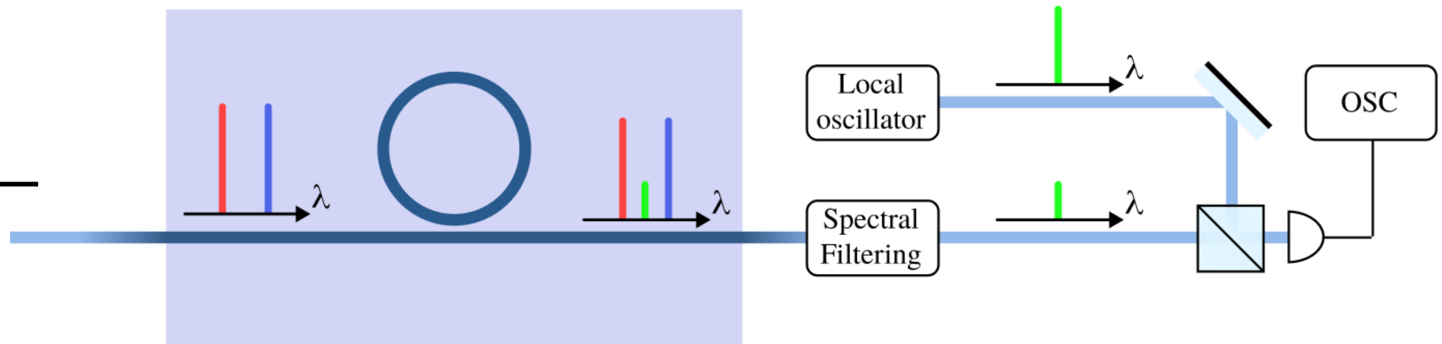

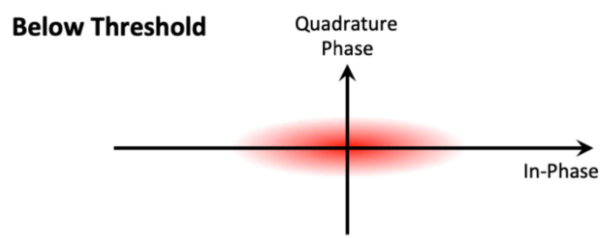

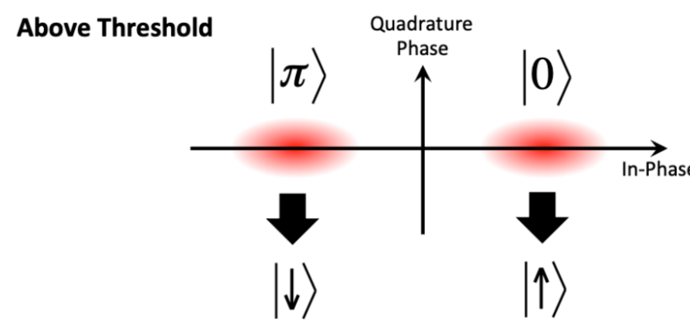

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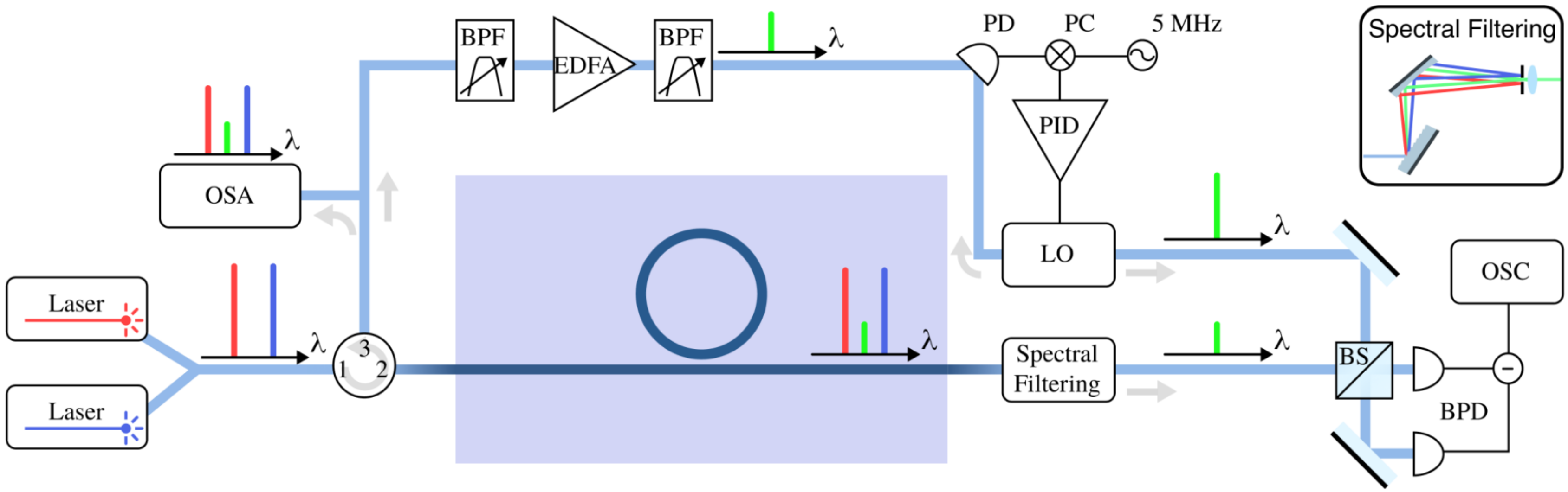

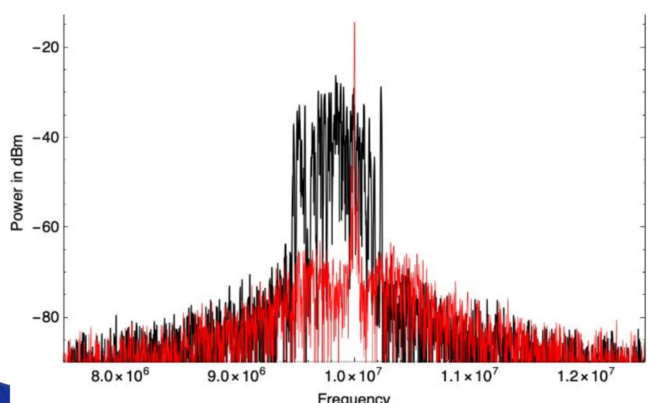

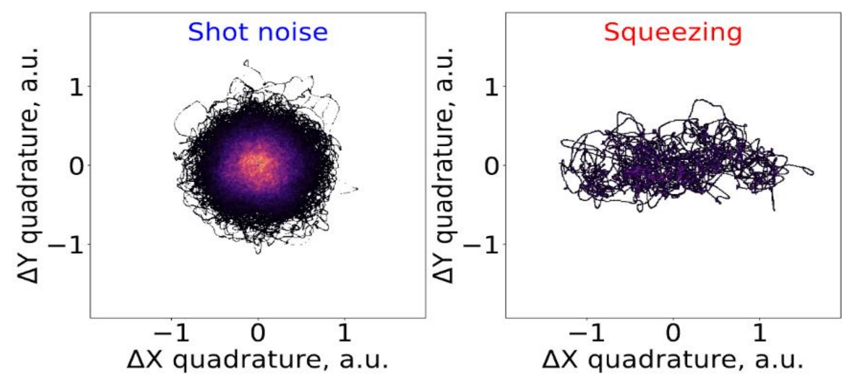

2 dB measured squeezing – corresponds to 5 dB in chip

Thanks for the attention!