GEOMETRICAL OPTIMIZATION OF BRISTLED WINGS OF MINIATURE FLAPPING FLIERS

Dmitry Kolomenskiy[1]*, Sergey E. Farisenkov[2]& Alexey A. Polilov[2]
[1]*Center for Materials Technologies, Skolkovo Institute of Science and Technology, Moscow, Russia*
[2]*Department of Entomology, Faculty of Biology, Lomonosov Moscow State University, Moscow, Russia.*

Summary We present a theoretical analysis that aims to explain the morphological features and allometric scalings observed in the bristled wings of miniature insects. We derive functional dependencies for parameters such as the number of bristles, the bristle length and diameter, the size of the central blade, etc., from the considerations of the wing inertia minimization under the aerodynamic and structural stiffness constraints.

INTRODUCTION

Many of the smallest insects possess bristled wings that superficially resemble bird feathers (see Fig. 1a), in contrast to the more common membranous wings. During the past decade, our understanding of the morphology and function of these bristled wings has significantly improved, see [1] for a review. In particular, it has been shown by an example of a featherwing beetle *Paratuposa placentis* that bristled wings generate sufficient aerodynamic force while consuming less power and exerting lesser inertial torques, in comparison with similarly sized membranous wings [2]. At the same time, it has been noticed that reconfiguration of the bristles (bending that reduces the aerodynamic drag) is avoided in the biological wings [3], which means that the minimization of wing mass is constrained by structural stiffness requirements. In this work, we find optimal values of the morphological parameters that minimize the wing mass under the constraints of ensuring sufficient bending stiffness to avoid reconfiguration and producing large enough aerodynamic force to keep the insect aloft. Concomitantly, we measured the morphological parameters of wings on a number of featherwing and membranous-wing beetle species to verify the model.

PROBLEM STATEMENT AND MODEL ASSUMPTIONS

We postulate that (i) the bristled wings should generate aerodynamic forces as large as those exerted on membranous wings of similar size, and (ii) that the bristles and the central blade must be sufficiently stiff to prevent reconfiguration. The wing moments of inertia are minimized via mass minimization under the two mentioned constraints.

The wing length R is treated as a fixed parameter and a proxy for the animal size. Let us assume geometrical similarity of wing and body shapes. In particular, $c \propto R$ and $L_b \propto R$, where L_b is the body length. Let us also assume that the angular motion of the wings is invariant. Then, the only kinematic parameter to be varied during the optimization is the wing beat frequency f.

We consider a simplified bristled wing model that consists of a circular flat blade and a fringe of bristles, see Fig. 1(b). The bristles are modeled as circular cylinders. It is assumed that all bristles are identical and have the sectional radius a. We deliberately neglect such features of the real bristles as the conical shape and secondary outgrowths. The planar shape is described by the following parameters, which are allowed to vary during the optimization process: radius of the central blade r; blade width h_p; bristle length l; number of the bristles N; distance from the wing hinge point to the center of the blade ξ. The wing length is therefore equal to $R = \xi + r + l$. Let $\hat{\cdot}$ denote distances non-dimensionalized by R. We then obtain $\hat{\xi} + \hat{r} + \hat{l} = 1$. The parameter ξ controls petiolation of the wings. In this study, we fix its value to $\hat{\xi} = 0.63$, which is representative of the small insect wings in general.

The characteristic velocity in this problem is the peak velocity at the center of the blade $U_\xi = 2\Phi f \xi \overline{\lambda}$, where $\overline{\lambda} = 1.7$ is the peak-to-average velocity ratio. The wing beat frequency f increases as the wing length decreases, but it has a peak and then the slope of $f(R)$ changes sign when the viscous drag becomes the dominant factor. The flapping amplitude in ptiliids is limited by clapping, therefore, we fix $\Phi = 180\,\mathrm{deg}$.

Wings must resist to the external deformation. In our model, we consider only the bending deformation and require that a wing does not bend excessively. A simple cantilever model is enough to describe the scaling of the wing center deflection y_ξ with the wing size parameters, $y_\xi = F_w \xi^3 / 3EJ_m$, where the aerodynamic loading is reduced to a point force F_w applied at distance ξ from the root, the material Young modulus is denoted as E, and the sectional second moment of area $J_m = K_m c_m h_m^3$ scaled in proportion with the membraneous part mean chord length c_m and the cube of the wing effective thickness h_m, with a constant proportionality coefficient K_m that we set equal to that of a constant rectangular cross-section, $K_m = 1/12$. It is customary to define the mean chord length as the area divided by the length of the wing. Here we use a similar definition, but based on the central blade parameters, $c_m = \pi r^2/(\xi + r)$.

* Corresponding author. E-mail: d.kolomenskiy@skoltech.ru

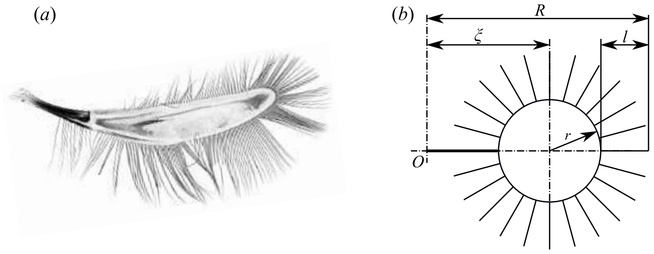

Figure 1. (*a*) Bristled wing of a featherwing beetle *Sindosium* sp., (*b*) Schematic drawing of the simplified bristled wing model. Point O is the wing shoulder joint.

We require that all optimized wings bend by a constant fraction of the wing length, $y_\xi = \hat{\gamma} R$, where $\hat{\gamma} = \arctan(8.1°)$. The equations are resolved with respect to h_m.

The wing mass $m_w = m_m + m_b$ is a sum of the mass of the central blade $m_m = \rho h_m \pi r^2$ and the mass of the bristles $m_b = N \rho_w l \pi a^2$. Recasting it in a form convenient for the optimization, we obtain

$$m_w = \pi \rho_w R^3 \left(\hat{h}_m (1 - \hat{\xi} - \hat{l})^2 + N \tilde{a}^2 \hat{l}^3 \right). \tag{1}$$

Smaller number of bristles, smaller diameter and larger length of each bristle relative to the wing length are desirable for the wing mass minimization. However, it is constrained by the requirement of sufficient aerodynamic force generation capacity. It is known from earlier research that an array of thin rods can produce almost the same aerodynamic force as a continuous solid plate in a flow at the Reynolds number $Re \sim O(10)$. To accomplish this, the gap between the bristles should be small enough to enable viscous blockage of the flow through it. Small enough means that each bristle should be immersed in the boundary layer of its neighbor. Based on the available morphometric data, we constrain the spacing-based Reynolds number to a constant value, $Re_d = U_\xi d/\nu \approx 0.51$, where d is the characteristic arc distance between the bristle centroids.

The bristles should be sufficiently stiff to sustain the action of aerodynamic force. If the bristles bend, the gap between them may vary. The bending deflection should be small enough for the gap to remain effectively the same as at rest. This condition can be formalized as follows. The deflection of the end of a bristle under the uniformly distributed load q is estimated using the linear beam theory as $y_b = ql^4/8EJ_b$, where the second moment of area is equal to $J_b = \pi a^4/4$. The maximum allowed deflection is set as a constant fraction of the gap, $y_b = \eta d$.

RESULTS AND DISCUSSION

Using the model described in the previous section, we solved numerically the constrained minimization problem for the wing mass. This has led us to the following preliminary conclusions.

- The bristled design minimizes the wing mass.
- The bristle diameter is large enough to guarantee sufficient bending stiffness.
- The bristle length is limited by the maximum aerodynamically allowed gap between bristles.
- Many of the trends emerging from the model are found consistent with the biological observation.
- The longest possible optimal bristled wing is $R \approx 3\mathrm{mm}$, beyond which the optimal bristles would be negligibly short and tightly packed.
- The shortest possible optimal bristled wing is $R \approx 0.1\mathrm{mm}$, beyond which the optimal bristled wing consists of only $N = 1$ bristle. For those regimes, flagellar locomotion seems more appropriate.

ACKNOWLEDGEMENTS

This work was supported by the Russian Science Foundation (Project 22-74-10010).

References

[1] Kolomenskiy D.: Recent Progress in the Biomechanics of Flight of Miniature Insects with Bristled Wings. *J. Phys. Soc. Jpn.* **92**:121004, 2023.

[2] Farisenkov S.E., Kolomenskiy D., Petrov P.N., Engels T., Lapina N.A., Lehmann F.-O., Onishi R., Liu H., Polilov A.A.: Novel Flight Style and Light Wings Boost Flight Performance of Tiny Beetles. *Nature* **602**:96–100, 2022.

[3] Lin Y.L., Pezzulla M., Reis P.M.: Fluid-Structure Interactions of Bristled Wings: the Trade-off Between Weight and Drag. *J. R. Soc. Interface* **20**:20230266, 2023.