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Effect of Detuning the Group Velocities of Optical Harmonics on the Reflection and Transmission of Radiation in an Active Periodic Medium

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Abstract—The authors study the effect detuning between the group velocities of two optical harmonics has on the reflection and transmission of radiation in a layered active quadratically nonlinear medium. The reflectance and transmittance at the fundamental and doubled frequencies are determined via numerical modeling. The soliton nature of energy transfer inside a medium is demonstrated.

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INTRODUCTION

New possibilities for exploiting the interaction between gain, loss, and forces that couple different optical components are always being discussed by researchers dealing with photonics [1, 2]. Such possibilities for generating, controlling, and transmitting light have emerged with the discovery of the real eigenfrequencies of non-Hermitian Hamiltonians that exhibit the parity–time (PT) symmetry.

Conservatism is violated in complex photonic structures with gain and loss. This opens up new prospects for using PT symmetry to generate optical signals with properties that go beyond conservative structures [3].

Layered media in which absorbing and generating elements alternate can be characterized by PT symmetry. We have already theoretically studied the propagation of optical radiation in an active layered medium with the quadratic nonlinearity. We used the approach developed in [4, 5], where results demonstrated the possibility of generating stable spatial gap solitons in media with the quadratic nonlinearity. Based on numerical modeling of a system of coupled Schrödinger equations for the envelopes of forward and backward waves at the fundamental and doubled frequencies, we obtained solitons of this type for active Bragg structures in particular [6–8]. We assumed that the values of the physical parameters (e.g., the Bragg coefficients of coupling and detuning from the Bragg resonance) lay in the same ranges as in [4, 5].

We also used numerical modeling in [9] to obtain results describing the reflective properties of a periodic medium, depending on the sign of asymmetric coupling.

In this work, we continue our research and study the effect the detuning of group velocities of waves has on second optical harmonic generation in layered active media.

FORMULATION OF THE PROBLEM

Let us write a system of coupled equations for normalized slowly changing amplitudes of forward and backward waves at fundamental frequency $E_{1\pm}$ (FF) and second harmonic frequency $E_{2\pm}$ (SH) [8, 9]:

$$i\left(\frac{\partial E_{1+}}{\partial \tau} + v_1\frac{\partial E_{1+}}{\partial z}\right) + D_{x,1}\frac{\partial^2 E_{1+}}{\partial x^2} + \delta_1 E_{1+} + (\kappa_1 + g_1)E_{1-} + \gamma_1 E_{1+}^* E_{2+} = 0, \tag{1}$$

$$i\left(\frac{\partial E_{1-}}{\partial \tau} - v_1\frac{\partial E_{1-}}{\partial z}\right) + D_{x,1}\frac{\partial^2 E_{1-}}{\partial x^2} + \delta_1 E_{1-} + (\kappa_1 - g_1)E_{1+} + \gamma_1 E_{1-}^* E_{2-} = 0, \tag{2}$$

$$i\left(\frac{\partial E_{2+}}{\partial \tau} + v_2\frac{\partial E_{2+}}{\partial z}\right) + D_{x,2}\frac{\partial^2 E_{2+}}{\partial x^2} + \delta_2 E_{2+} + (\kappa_2 + g_2)E_{2-} + \gamma_2 E_{1+}^2 = 0, \tag{3}$$

$$i\left(\frac{\partial E_{2-}}{\partial \tau} - v_2\frac{\partial E_{2-}}{\partial z}\right) + D_{x,2}\frac{\partial^2 E_{2-}}{\partial x^2} + \delta_2 E_{2-} + (\kappa_2 - g_2)E_{2+} + \gamma_2 E_{1-}^2 = 0. \tag{4}$$

Here, $E_{1\pm} = \frac{E_1^{\pm}}{\sqrt{I_{10}}}$, $E_{2\pm} = \frac{E_2^{\pm}}{\sqrt{I_{10}}}$ are the slowly varying dimensionless amplitudes of interacting waves, normalized to the square root of the incident radiation's peak intensity I_{10}. Evolution coordinate τ is dimensionless time, propagation occurs along longitudinal coordinate z with group velocities $v_{1,2}$ for the fundamental and doubled frequencies, and x is a transverse coordinate. Parameters $\delta_1 = \left(k_1 - \frac{\pi}{d}\right)\frac{1}{|\kappa|}$, $\delta_2 = \left(k_2 - \frac{2\pi}{d}\right)\frac{1}{|\kappa|}$ are normalized detunings from the Bragg resonance at the fundamental and double frequencies, where $\kappa = \frac{\omega_0}{c\sqrt{\varepsilon_0(\omega_0)}}\frac{\Delta\varepsilon_R}{4}$ is the parameter of Bragg coupling between counterpropagating waves at the fundamental frequency; ω_0 is the carrier frequency of the fundamental harmonic radiation; k_1 and k_2 are the wave numbers of radiation at the fundamental and double frequencies, respectively; and d is the period of the layered structure. Dimensionless parameters $\kappa_1 = \frac{\kappa}{|\kappa|}$ and $\kappa_2 = \frac{2\omega_0}{c\sqrt{\varepsilon_0(2\omega_0)}}\frac{\Delta\varepsilon_{R2}}{4|\kappa|}$ describe the Bragg coupling between counterpropagating waves at the fundamental and SH frequencies, respectively, and $g_1 = \frac{\omega_0}{c\sqrt{\varepsilon_0}}\frac{\Delta\varepsilon_I}{4}\frac{1}{|\kappa|}$ and $g_2 = \frac{2\omega_0}{c\sqrt{\varepsilon_0(2\omega_0)}}\frac{\Delta\varepsilon_{I2}}{4}\frac{1}{|\kappa|}$ are the corresponding dimensionless parameters of the asymmetric coupling between counterpropagating waves. The medium is passive at $g_1 = g_2 = 0$; otherwise, it is active. Parameters $\gamma_1 = \frac{4\pi}{c^2 k_1}\omega_0^2\chi^{(2)}(-\omega_0, 2\omega_0)\frac{\sqrt{I_{10}}}{|\kappa|}$ and $\gamma_2 = \frac{2\pi}{c^2 k_2} \times 4\omega_0^2\chi^{(2)}(\omega_0, \omega_0)\frac{\sqrt{I_{10}}}{|\kappa|}$ characterize the quadratic nonlinearity, where $\chi^{(2)}$ is the nonlinear susceptibility of the medium. We limit our consideration to wide beams, set $D_{x,1} = D_{x,2} = 0$ in (1)–(4), and ignore the transverse intensity distribution to consider only the profile along longitudinal coordinate z.

Let the specified nonlinear periodic structure be bound by z. Its left and right boundaries are then $z = L_{\text{left}}$ and $z = L_{\text{right}}$, and it is surrounded by a linear medium. The dynamics of forward and backward waves inside the nonlinear medium is described by system (1)–(4). Outside the medium, it is described by transfer equations $\frac{\partial E_{1,2\pm}}{\partial\tau} \pm v_{1,2}\frac{\partial E_{1,2\pm}}{\partial z} = 0$. A beam of the fundamental frequency strikes the medium from the left. It is set at the initial moment of time $\tau = 0$ on a forward wave of the fundamental frequency with center L_0 to the left of the left boundary of the medium ($L_0 < L_{\text{left}}$), in the form of a soliton profile of characteristic width h_z:

$$E_{1+}(z, \tau = 0) = \cosh^{-1}\left((z - L_0)/h_z\right).$$

The remaining waves carry no signals at the initial moment of time: $E_{1-}(z, \tau = 0) = E_{2+}(z, \tau = 0) = E_{2-}(z, \tau = 0) = 0$.

The propagation of two-color radiation in an infinite active quadratic layered medium described by Eqs. (1)–(4) satisfies the law of a change in energy:

$$\begin{aligned}\frac{\partial}{\partial\tau}\int_{-\infty}^{+\infty}\left(\gamma_2\left(|E_{1+}|^2 + |E_{1-}|^2\right) + \gamma_1\left(|E_{2+}|^2 + |E_{2-}|^2\right)\right)dz \\ + 4g\gamma_2\int_{L_{\text{left}}}^{L_{\text{right}}}\operatorname{Im}\left(E_{1+}^{*}E_{1-}\right)dz \\ + 4g_2\gamma_1\int_{L_{\text{left}}}^{L_{\text{right}}}\operatorname{Im}\left(E_{2-}E_{2+}^{*}\right)dz = 0.\end{aligned} \tag{5}$$

In a passive medium, law (5) ($g_1 = g_2 = 0$) changes to the law of energy conservation in [4].

Equations (1)–(4) are solved numerically using a conservative nonlinear difference scheme with a matrix sweep and an iteration algorithm. Calculations are made in the ranges of $0 \le z \le L_z$ and $0 \le \tau \le L_\tau$ under zero boundary conditions specified on the left boundary of forward waves and the right boundary of backward waves:

$$\begin{aligned}E_{1+}(z = 0, \tau) = E_{2+}(z = 0, \tau) \\ = E_{1-}(z = L_z, \tau) = E_{2-}(z = L_z, \tau) = 0.\end{aligned}$$

This ensures that no additional energy enters the system over time. No additional conditions are specified on the opposite boundaries of the computational domain, since the transfer equation is solved on them and energy freely leaves the system without reflection. Since law (5) was obtained under a condition of infinite space, we changed it in accordance with the considered limited computational domain:

$$\begin{aligned}\frac{\partial}{\partial\tau}\int_{0}^{L_z}\left(\gamma_2\left(|E_{1+}|^2 + |E_{1-}|^2\right) + \gamma_1\left(|E_{2+}|^2 + |E_{2-}|^2\right)\right)dz \\ + 4g\gamma_2\int_{L_{\text{left}}}^{L_{\text{right}}}\operatorname{Im}\left(E_{1+}^{*}E_{1-}\right)dz \\ + 4g_2\gamma_1\int_{L_{\text{left}}}^{L_{\text{right}}}\operatorname{Im}\left(E_{2-}E_{2+}^{*}\right)dz \\ + \gamma_2 v_1\left[|E_{1+}(L_z, \tau)|^2 + |E_{1-}(0, \tau)|^2\right] \\ + \gamma_1 v_2\left[|E_{2+}(L_z, \tau)|^2 + |E_{2-}(0, \tau)|^2\right] = 0.\end{aligned} \tag{6}$$

The last four terms correspond to the release of energy from the system of each forward and backward wave of the fundamental and doubled frequencies. Considering them separately allows us to follow which waves carry the energy away.

To determine properties of the reflection and transmission of radiation in an active layered medium, we calculate the reflectance and transmittance at the fundamental and doubled frequencies for moment of time L_τ in the form of an integral of the intensities passed through the medium, allowing for the energy that has gone beyond the boundary of the calculated region:

$$R_1 = \frac{\int_0^{L_{\text{left}}} \left|E_{1-}(z, L_\tau)\right|^2 dz + v_1 \int_0^{L_\tau} \left|E_{1-}(0,\tau)\right|^2 d\tau}{\int_0^{L_{\text{left}}} \left|E_{10}(z)\right|^2 dz}, \quad (7)$$

$$T_1 = \frac{\int_{L_{\text{right}}}^{L_z} \left|E_{1+}(z, L_\tau)\right|^2 dz + v_1 \int_0^{L_\tau} \left|E_{1+}(L_z,\tau)\right|^2 d\tau}{\int_0^{L_{\text{left}}} \left|E_{10}(z)\right|^2 dz}, \quad (8)$$

$$R_2 = \frac{\int_0^{L_{\text{left}}} \left|E_{2-}(z, L_\tau)\right|^2 dz + v_2 \int_0^{L_\tau} \left|E_{2-}(0,\tau)\right|^2 d\tau}{\int_0^{L_{\text{left}}} \left|E_{10}(z)\right|^2 dz}, \quad (9)$$

$$T_2 = \frac{\int_{L_{\text{right}}}^{L_z} \left|E_{2+}(z, L_\tau)\right|^2 dz + v_2 \int_0^{L_\tau} \left|E_{2+}(L_z,\tau)\right|^2 d\tau}{\int_0^{L_{\text{left}}} \left|E_{10}(z)\right|^2 dz}. \quad (10)$$

These coefficients characterize both the reflecting and transmitting properties of an active layered medium at the fundamental frequency, and the ability to generate the SH in the direction of a pumping drop and the one opposite it. When it is linear ($\gamma_1 = \gamma_2 = 0$) at the precise Bragg resonance ($\delta_1 = 0$) in a passive medium ($g_1 = 0$) of infinite length ($L_{\text{right}} = \infty$), an incident wave is fully reflected ($R_{1,\infty} = 1$, $T_{1,\infty} = 0$) and the SH does not arise. If the medium is active, an incident signal can be amplified ($R_{1,\infty} > 1$ at $g_1 < 0$) or weakened ($R_{1,\infty} < 1$ at $g_1 > 0$), depending on the sign of g_1, and there is no transmitted wave ($T_{1,\infty} = 0$) [9]. Upon detuning from the Bragg resonance ($\delta_1 \neq 0$), the reflectance changes and a nonzero transmitted wave appears ($T_1 > 0$). Quadratic nonlinearity results in the incident wave generating the SH in a medium for which the conditions of Bragg resonance differ from the fundamental frequency. This seriously complicates the picture, and nonzero T_2 and R_2 arise. Under certain conditions, the incoming signal and the generated SH form a two-color soliton [4, 5, 8] that propagates toward the right boundary in a layered nonlinear medium, where it is reflected and the energy partially leaves the medium. The reflected signal moves back to the left boundary of the medium in a soliton-like manner.

The formation of a soliton is affected by the detuning of group velocities. A soliton does not form if the velocities of the waves of the fundamental and doubled frequencies differ appreciably. This process seriously affects the reflecting and transmitting properties of the layered nonlinear medium.

RESULTS FROM NUMERICAL MODELING

Numerical modeling of Eqs. (1)–(4) was done to study the dependences of reflectance and transmittance on group velocity detuning. We considered a case where the fundamental frequency was near Bragg resonance $\delta_1 = -0.9$ and the SH was far from the Bragg resonance: $\delta_2 = 5$. The parameters of our calculations were $\kappa_1 = 1$, $\kappa_2 = 0$, $\gamma_1 = 1$, $\gamma_2 = 0.5$, and an initial beam width of $h_z = 5$, which corresponds to incident wave peak intensity $I_{10} = 500$ MW/cm^2, $\kappa = 0.5$ mm^{-1}, and $\frac{4\pi}{c^2 k_1}\omega_0^2\chi^{(2)}(-\omega_0, 2\omega_0) \approx 10^{-3}$ W$^{-1/2}$ [4, 5]. The sizes of the computational domain were $L_z = 120$ and $L_\tau = 120$, the boundaries of the layered medium were $L_{\text{letf}} = 60$ and $L_{\text{right}} = 90$, and the beam's initial position was $L_0 = 30$. SH group velocity $v_2 = 1$ was fixed, and the fundamental frequency's group velocity varied in the range of $0.5 \le v_1 \le 2$.

A passive medium with $g_1 = 0$ and $g_2 = 0$ was investigated first. Figure 1a shows the dependences of the reflectance and transmittance on the group velocity of the fundamental frequency. The transmittance of the fundamental frequency has a pronounced maximum near $v_1 = 1$, indicating there is stronger transmission when the group velocities are equal. Note too that reflectance at the SH rises as v_1 falls, so it is logical to use such a medium as the source of SH waves at group velocities of the fundamental frequency that are much lower than at the doubled frequency. Figure 1b shows the temporal dependences of the peak intensities and beam widths at $v_1 = 1$. The initial beam propagates with minor changes until around $\tau = 30$. Part of it then enters the medium, and part is reflected. It becomes more intense, its width shrinks, and the SH is generated. The beam profile at $\tau = 60$ is presented in Fig. 1c. There is also an SH peak at the maximum fundamental frequency, showing it is a two-color

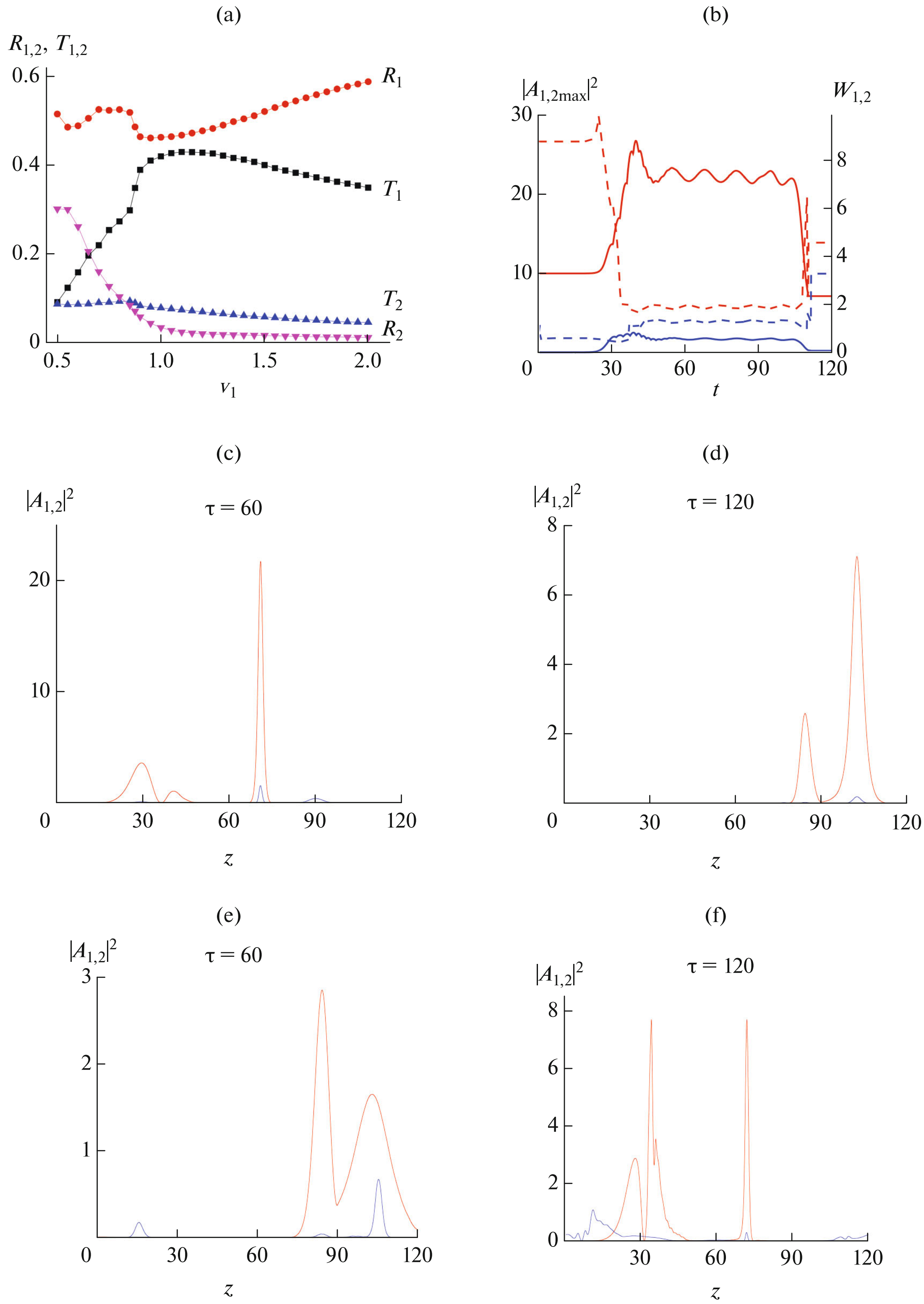

Fig. 1. (a) Reflection and transmission of waves in a passive medium. $\kappa_1 = 1$, $\kappa_2 = 0$, $g_1 = 0$, $g_2 = 0$, $\delta_1 = -0.9$, $\delta_2 = 5$, and $v_2 = 1$. Reflectances $R_{1,\,2}$ and transmittances $T_{1,\,2}$ of the fundamental frequency and second harmonic depending on the group velocity of a wave with fundamental frequency v_1. (b) Maximum intensities (solid line) and widths (dashed line) of the fundamental frequency (red) and the second harmonic (blue) at $v_1 = 1$. Beam profiles of the fundamental frequency (red) and the second harmonic (blue) at (c, d) $v_1 = 1$, (e) 2, and (f) 0.5 at different moments of time τ.

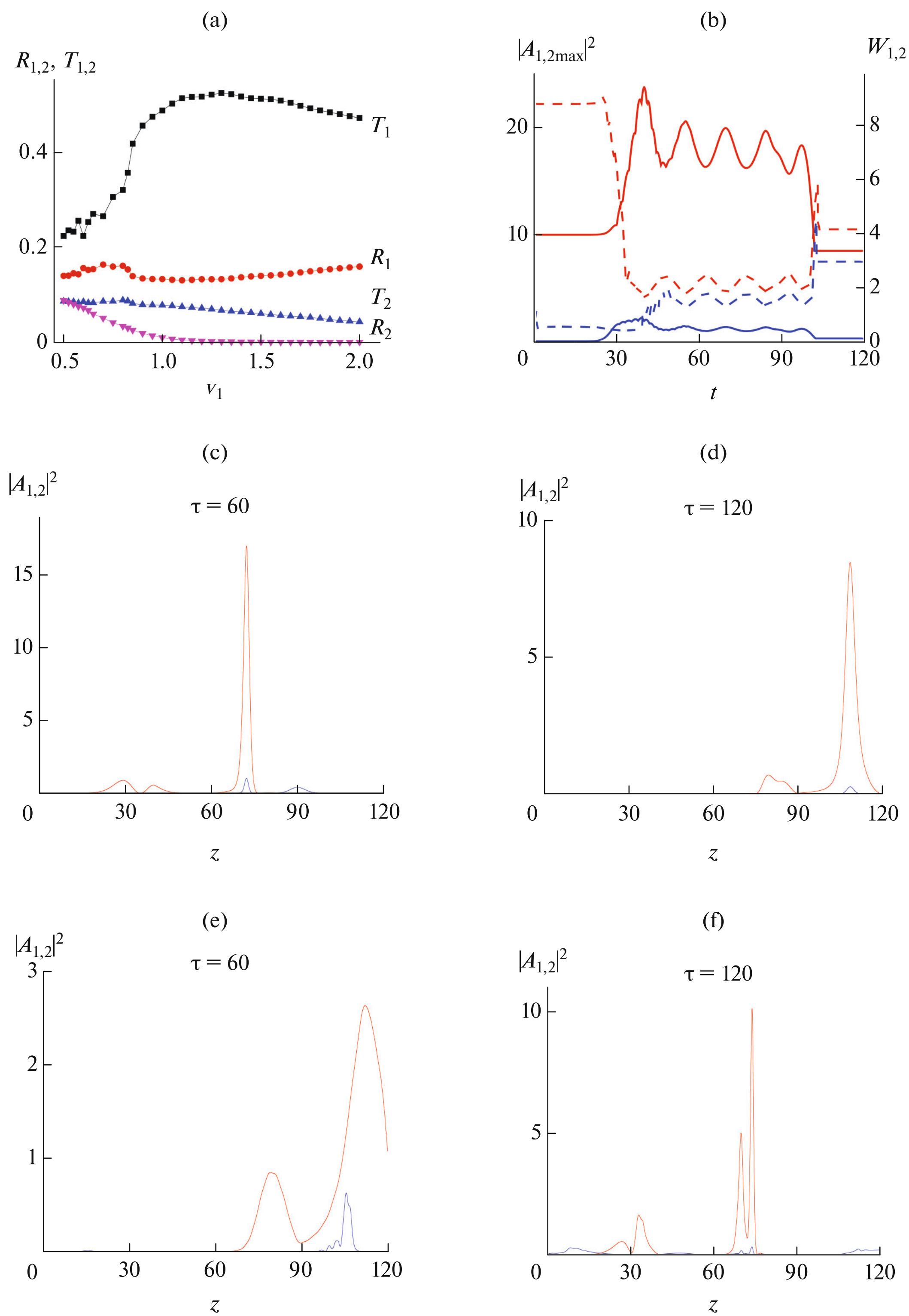

Fig. 2. Reflection and transmission of waves in an active medium. Similar to Fig. 1, except for $g_1 = 0.5$.

soliton that moves with some oscillations in the medium up to around $\tau = 100$. The intensity then falls sharply, due to most of the energy being released from the medium. Figure 1g shows the beam profile at $\tau = 120$, when some energy remains the medium and some has left.

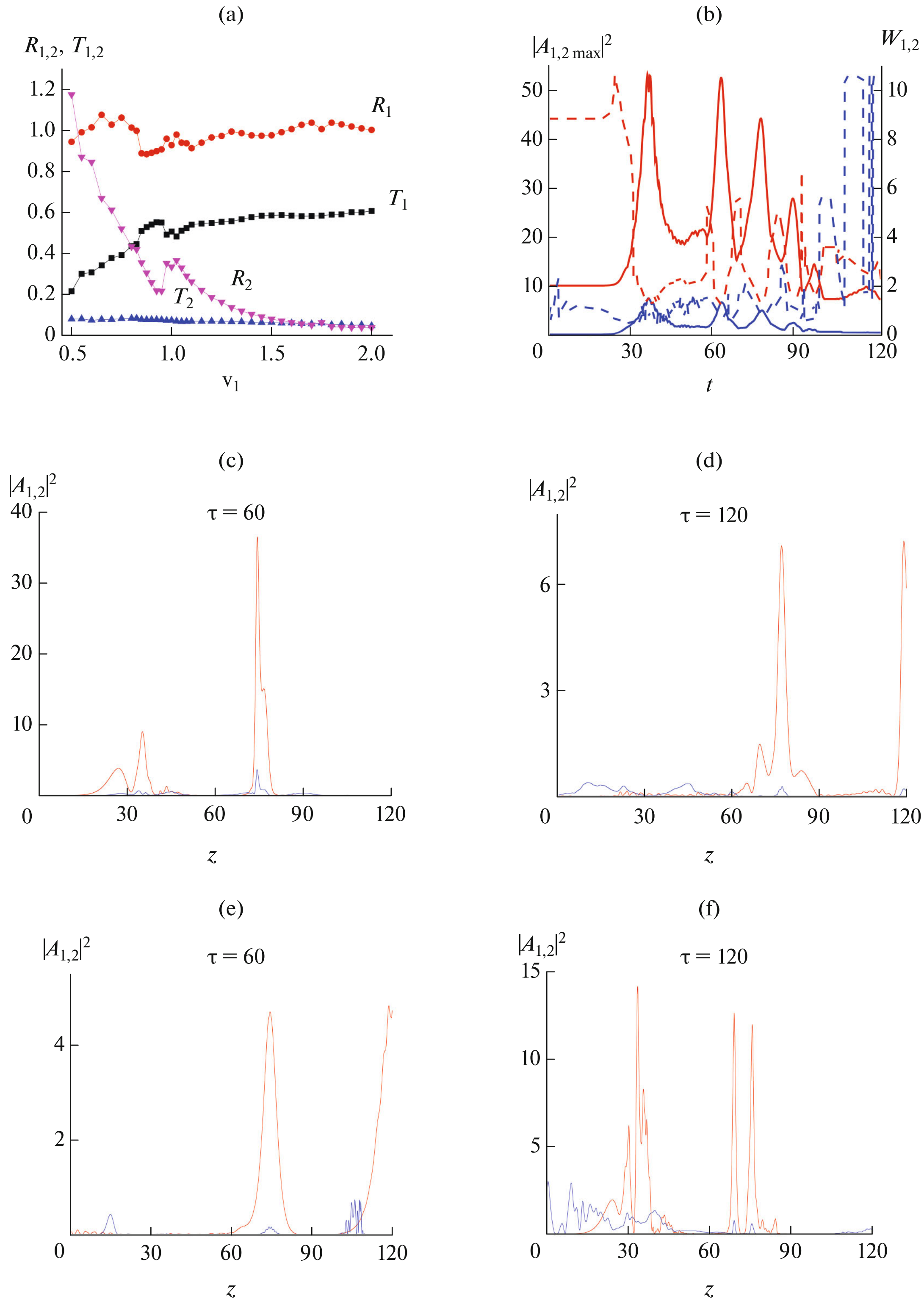

Fig. 3. Reflection and transmission of waves in an active medium. Similar to Fig. 1, except for $g_1 = -0.5$.

Figure 1e shows the beam profile at $v_1 = 2$ and $\tau = 60$, when part of the energy has left the medium and some remains inside. The part that has left is similar to a two-color soliton, but it is considerably wider and the centers of the beams of the first and second harmonics have shifted noticeably. This is explained by the different velocities of the radiation at the fundamental and doubled frequencies in a linear medium.

Figure 1d shows the profiles for $v_1 = 0.5$ at $\tau = 120$. Part of the energy reflected to the left of the medium can be seen, and a soliton is observed in the medium itself. This shows the soliton nature of energy transfer within a medium is widespread at different parameters. The soliton velocity depends on v_1: the higher the rate of radiation at the fundamental frequency in a linear medium, the higher the velocity of the soliton in a nonlinear medium.

Figure 2 illustrates the case of an active medium with $g_1 = 0.5$, similar to the one discussed above. The intensity of the fundamental frequency falls. The reflectance has a maximum near $v_1 = 1$ whose value is close to that of a passive medium. The remaining coefficients are several times lower than in a passive medium. The temporal dependence of the peak intensities at $v_1 = 1$ and beam profiles at different moments of time also indicate the transfer of energy in the medium is of a soliton nature. In contrast to a passive medium, the reflection of energy at the fundamental frequency is much lower than this passage through a nonlinear medium. At the same time, the reflectances and transmittances of the SH energy are similar to the corresponding values for a passive medium.

Figure 3 presents the case of an active medium with $g_1 = -0.5$; the intensity of the waves grows. The SH reflectance grows sharply as v_1 falls. The beam profiles show that several oscillating solitons form and the energy is pumped between them. The picture is therefore quite complex, as can be seen from the plots of the peak intensity. We can generally say that such a medium can be used to generate SH radiation directed toward the incident radiation when the group velocity of radiation at the fundamental frequency is notably lower than at the doubled frequency.

CONCLUSIONS

We investigated dependences of the reflective properties of nonlinear active layered media on the detuning of group velocities between waves of the fundamental frequency and second harmonic. The magnitudes of reflectance and transmittance were determined. The soliton nature of energy transfer inside such a medium was demonstrated. Based on this, materials can be selected for specific frequencies in order to create active filters and generate signals of doubled frequency.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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