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Guaranteeing Estimation Method for Determining Failures in a Redundant Sensor Unit

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Abstract—An algorithm is proposed to determine failures for a redundant inertial sensor unit by means of the guaranteeing estimation method. The numerical testing confirms the efficiency of the proposed algorithm.

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1. INTRODUCTION

To improve the reliability of navigation systems, researchers apply redundant inertial sensor units, that is, angular velocity sensors and accelerometers. Instead of three mutually orthogonal sensor elements (in each unit), six sensors are used. It is assumed that, at one time instance, no more than two failures may occur in all channels of each unit. We need to detect possible failures in the measurement channels of the units. In other words, we need to determine the presence of failure and find out in which channel it occurred.

For failure detection the nature of abnormal errors in the readings of sensors is insignificant. The actions to detect and localize failures are identical for both units of sensor elements. Therefore, in this work we consider only the unit of angular velocity sensors.

We propose the guaranteeing approach to determine failures the advantage of which is that the failure detection problem is posed as the optimal estimation problem. In addition to that, we succeed in computing the optimal guaranteed error of determining the corresponding parameters.

2. FORMULATION OF THE ORIGINAL PROBLEM

Let measurements delivered by a unit of angular velocity sensors have the form (at a given time instance):

$$z = Gq + \varrho, \tag{1}$$

where $z = (z_1, \dots, z_6)^{\mathsf{T}} \in \mathbb{R}^6$ is a vector composed of readings of the unit of angular velocity sensors; $q \in \mathbb{R}^3$ is the vector of estimated angular velocity components in projections onto the instrument frame; $\varrho = (\varrho_1, \dots, \varrho_6)^{\mathsf{T}} \in \mathbb{R}^6$ is the vector composed of measurement uncertainties.

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The matrix G is given by the relation

$$G = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_6 \end{pmatrix} = \begin{pmatrix} -\cos\varphi & -\sin\varphi & 0 \\ \cos\varphi & \frac{1}{2}\sin\varphi & -\frac{\sqrt{3}}{2}\sin\varphi \\ -\cos\varphi & \frac{1}{2}\sin\varphi & \frac{\sqrt{3}}{2}\sin\varphi \\ \cos\varphi & -\sin\varphi & 0 \\ -\cos\varphi & \frac{1}{2}\sin\varphi & -\frac{\sqrt{3}}{2}\sin\varphi \\ \cos\varphi & \frac{1}{2}\sin\varphi & \frac{\sqrt{3}}{2}\sin\varphi \end{pmatrix},$$

where the angle φ is defined by the equalities $\sin\varphi = \sqrt{\frac{2}{3}}$, $\cos\varphi = \frac{1}{\sqrt{3}}$.

The original problem is to establish the presence of failure (failures) and to determine where it occurred.

3. PROBLEM OF GUARANTEEING ESTIMATION

Firstly, we consider an auxiliary problem of guaranteeing parameter estimation [1–6] in the presence of two failures in measurement errors.

As above, suppose that the measurements are given by formula (1). We need to estimate the scalar value $l = a^{\mathsf{T}}q$, where $a \in \mathbf{R}^3$ is a given vector.

Let two failures be allowed in the measurement errors. Then the model of measurement errors is given by

$$\varrho \in \mathcal{P}, \quad \mathcal{P} = \bigcup_{\omega\in\Omega} \mathcal{P}(\omega),$$

$$\mathcal{P}(\omega) = \left\{ \varrho \in \mathbf{R}^6 \,\middle|\, |\varrho_i| \leqslant \sigma,\ i = 1,\dots,6,\ i \notin \omega \right\},$$

where Ω is the set of all pairs $\omega = \{i_1, i_2\}$ $(i_1 < i_2)$ from the set $\{1,\dots,6\}$ and σ is a given positive value characterizing the maximum measurement error without failures.

Note that the set $\mathcal{P}$ is nonconvex and unbounded, in contrast to the classical formulations of the problems of guaranteeing estimation of parameters, where $|\varrho_i| \leqslant \sigma$. For instance, in the case of three measurements and one failure, this set is a sum of mutually orthogonal prisms in $\mathbf{R}^3$.

Consider all possible estimators $\hat{l} = s(z)$ for $l = l(q)$. We also introduce the set of the parameters $q \in \mathbf{R}^3$ consistent with a given vector z:

$$Q(z) = \left\{ q \in \mathbf{R}^3 \,\middle|\, Gq = z - \varrho,\ \varrho \in \mathcal{P} \right\}.$$

Note that the set $Q(z)$ can be disconnected, that is, can consist of isolated parts.

The problem of (a posteriori, that is, for a given z) guaranteeing estimation is in finding the estimator $s^0(z)$ such that for all other estimators $s(z)$ it is true that

$$\max_{q\in Q(z)} |l(q) - s^0(z)| \leqslant \max_{q\in Q(z)} |l(q) - s(z)|.$$

The minimum and maximum admissible values of l are determined by solving the following extremal problems:

$$l_{\min} = \min_{q\in Q(z)} a^{\mathsf{T}}q, \quad l_{\max} = \max_{q\in Q(z)} a^{\mathsf{T}}q.$$

It is clear that the optimal guaranteeing a posteriori estimator is given by

$$s^0(z) = \frac{1}{2}\left(l_{\max} + l_{\min}\right) \tag{2}$$

and the a posteriori optimal guaranteeing estimation error is determined by the formula

$$\delta^0(z) = \max_{q \in Q(z)} |l(q) - s^0(z)| = \frac{1}{2}(l_{\max} - l_{\min}). \tag{3}$$

Let us discuss the procedure for computing $l_{\min}$ and $l_{\max}$. At failures in measurements with indices from ω, we introduce the set of the vectors $q \in \mathbf{R}^3$ that are consistent with the vector z:

$$Q_\omega(z) = \left\{ q \in \mathbf{R}^3 \,\middle|\, |z_i - G_i q| \leqslant \sigma,\ i = 1, \dots, 6,\ i \notin \omega \right\}.$$

Then

$$Q(z) = \bigcup_{\omega \in \Omega} Q_\omega(z), \quad l_{\max} = \max_{\omega \in \Omega} I_\omega(z), \quad l_{\min} = \min_{\omega \in \Omega} J_\omega(z),$$

$$I_\omega(z) = \max_{q \in Q_\omega(z)} a^\mathsf{T} q, \quad J_\omega(z) = \min_{q \in Q_\omega(z)} a^\mathsf{T} q. \tag{4}$$

The problems dual to problems (4) [6, 7] are given by

$$I_\omega(z) = \min_{X_\omega} f(x), \quad f(x) = \sum_{i=1}^{6} (\sigma |x_i| + z_i x_i), \tag{5}$$

$$J_\omega(z) = -\min_{X_\omega} g(x), \quad g(x) = \sum_{i=1}^{6} (\sigma |x_i| - z_i x_i), \tag{6}$$

$$X_\omega = \left\{ x \in \mathbf{R}^6 \,\middle|\, \sum_{i=1}^{6} G_i^\mathsf{T} x_i = a,\ x_j = 0,\ j \in \omega \right\}. \tag{7}$$

Then the optimal estimator $s^0(z)$ and the optimal estimation error $\delta^0(z)$ are determined by formulas (2) and (3).

It is easy to show that problems (5)–(7) are equivalent to the linear programming problems if x_i is replaced with $u_i - v_i$ and $|x_i|$ is replaced with $u_i + v_i$, $u_i, v_i \geqslant 0$, $i = 1, \dots, 6$. Therefore, they can be efficiently solved, for instance, by the simplex method [7] or by the interior point method [8].

To compute $l_{\max}$ and $l_{\min}$, we need to solve at most $2C_6^2 = 30$ linear programming problems.

4. REDUCTION OF THE FAILURE DETECTION PROBLEM TO ESTIMATION PROBLEM

To solve the failure detection problem, we consider it to be the estimation problem. To estimate the components of the vector of measurement error ϱ_i, $i = 1, ..., 6$, we propose to solve 6 problems of optimal guaranteeing estimation for the parameter $l = a^\mathsf{T} q$ at $a = G_i$, where G_i are the rows of the matrix G. After that, by obtaining the estimate for $G_i q$, we find the estimate for the measurement error ϱ_i by the formulas (see (2) and (3)):

$$\hat{\varrho}_i = z_i - s_0^i(z), \quad \text{where} \quad s_0^i(z) = s_0(z) \quad \text{at} \quad a = G_i, \quad i = 1, ..., 6, \tag{8}$$

and the estimation errors are

$$\widehat{\delta \varrho_i} = \delta_0^i(z), \quad \text{where} \quad \delta_0^i(z) = \delta_0(z) \quad \text{at} \quad a = G_i, \quad i = 1, ..., 6. \tag{9}$$

Next, we check the condition

$$|\widehat{\varrho}_i \pm \widehat{\delta \varrho_i}| \geqslant \Delta, \quad \text{where } \Delta \text{ is a given threshold.} \tag{10}$$

From inequality (10) we establish the fact of failure and the index of the failed channel. Thus, the problem of detecting two failures reduces to solving at most $30 \times 6 = 180$ linear programming problems.

In the numerical solution of the variational problems (5)–(7), due to their small dimension, the corresponding computational load is rather moderate. In any case the optimal guaranteeing estimation problem formulated here can serve as the reference problem for estimating the quality of functioning of other (onboard) algorithms that can be simpler to implement. We emphasize that the guaranteeing approach also allows obtaining the optimal estimation error.

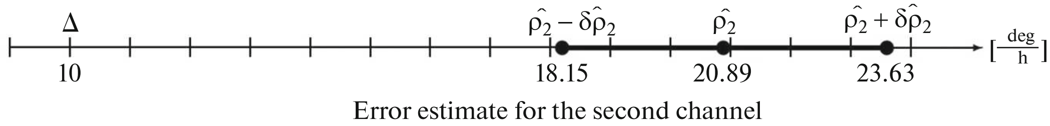

Fig. 1. Estimation error for the second channel.

5. NUMERICAL TESTING OF THE ALGORITHM

In this work we performed testing of the proposed algorithm for failure detection in the measurements obtained by means of an simulation program.

We chose the threshold value equal to $\Delta = 10$ deg/h. We assumed the estimates of measurement errors ϱ_i whose absolute value is larger than Δ to be abnormal. Everywhere below, as the unit of measurement we use deg/h.

In our testing we prescribed the following value of the vector of parameters q:

$$q = (-172.82,\ 604.19,\ -1284.63)^{\mathsf{T}}.$$

In our modeling we took the upper bound of absolute values for the regular (nonfailed) fluctuation measurement error σ to be equal to 1 deg/h.

The values of the fluctuation errors are given by

$$\varrho^f = (0.50,\ 0.10,\ -0.80,\ -0.01,\ -0.02,\ 0.30)^{\mathsf{T}}.$$

Suppose that failures in the second and third measurements occurred simultaneously: $\varrho_2 = 20 + \varrho_2^f$ and $\varrho_3 = -50 + \varrho_3^f$, that is, the failures are equal to 20 and -50, respectively. Taking into account the introduced values of q and ϱ, the measurements computed by formula (1) are given by

$$z = (-393.04,\ 1085.35,\ -612.73,\ -593.11,\ 1254.79,\ 761.19)^{\mathsf{T}}.$$

We solved the linear programming problems (5)–(7) in MATLAB by means of the linprog function that implements the interior point method [8].

For instance, in solving problems (5)–(7), for the second channel we obtained that for all values of ω containing at least one index of failed channel, the values of the objective functions $I_\omega(z)$ and $J_\omega(z)$, are $-\infty$ and ∞, respectively. This reflects the case when the corresponding set $Q_\omega(z)$ is empty. The functionals took on finite values for the only pair $\omega = (2, 3)$ containing the indices of the failed measurement channels. The values $l_{\min}$ and $l_{\max}$ are 1051.72 and 1057.20, respectively. Similar computations were also carried out for other measurement channels.

The estimates of residues obtained by formulas (8) are

$$\widehat{\varrho} = (-0.00,\ 20.89,\ -51.35,\ -0.00,\ -0.00,\ -0.00)^{\mathsf{T}}.$$

The corresponding estimation errors obtained by formulas (9) are determined by the equality

$$\widehat{\delta\varrho} = (1.00,\ 2.74,\ 2.74,\ 1.00,\ 1.00,\ 1.00)^{\mathsf{T}}.$$

After checking condition (10), we established that failures were detected in the second and third measurement channels.

Figure 1 presents the results of computations for the second channel. We can see that the entire segment lies to the right from the threshold, and condition (10) is met and the failure is detected.

For the third channel the corresponding figure is similar to Fig. 1, and the guaranteed confidence interval also completely lies beyond the threshold value Δ. For other nonfailed channels the corresponding confidence intervals lie to the left from the threshold.

Note that applying the failure detection methods based on the classical estimation methods (the least-squares method and the least absolute deviation method) for such a small number of measurements (six measurements) yields no advantages against the guaranteeing estimation method. Using the numerical modeling we established that the least-squares method does not identify even a single failure and the least absolute deviation method detects a single failure, but does not identify two failures. In addition, unlike the guaranteeing approach, using the traditional methods does not allow obtaining guaranteed confidence intervals for the estimates.

6. CONCLUSIONS

In this work we solved the problem of failure detection in a redundant unit of inertial sensors using the guaranteeing estimation method. The proposed algorithm allows not only establishing the presence of two simultaneous failures, but also obtaining the guaranteed confidence intervals for failure estimation.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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